ASSIGNMENT
CH-MATRICES

Q1. If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ **, find *x,y,z,w.***

(ANS: $x=1, y=2, z=3, w=4$ **)**

Q2. Find the value of *x, y ,a,* and *b* if

$$\begin{bmatrix} 2x-3y & a-b & 3 \\ 1 & x+4y & 3a+4b \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$$

(ANS: $x=2, y=1, a=3, b=5$ **)**

Q3. For what values of x and y are the following matrices equal?

$$A = \begin{bmatrix} 2x+1 & 2y \\ 0 & y^2-5y \end{bmatrix}, \quad B = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

Q4. If $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c-2 \\ 2b+4 & -21 & 0 \end{bmatrix}$ **.Obtain the values of *a, b, c, x, y,* and *z.***

(ANS: $a=-2, b=-7, c=-1, x=-3, y=-5, z=2$ **)**

Q5. Give an example of

(i) a row matrix which is also a column matrix,

(ii) a diagonal matrix which is not scalar,

(iii) a triangular matrix

Q6. Construct a 2×3 **matrix whose elements** a_{ij} **are given by**

(i) $a_{ij} = \frac{(i+j)^2}{2}$ **(ii)** $a_{ij} = \frac{(i-j)^2}{2}$ **(iii)** $a_{ij} = \frac{(i-2j)^2}{2}$ **(iv)** $\frac{|2i-3j|}{2}$

(v) $a_{ij} = \frac{|-3i+j|}{2}$

Q7. Construct a 4×3 **matrix whose elements** a_{ij} **are given by**

(i) $a_{ij} = \frac{i-j}{i+j}$ **(ii)** $a_{ij} = i$ **(iii)** $a_{ij} = 2i + \frac{i}{j}$

(ANS: $\begin{bmatrix} 0 & -1/3 & -1/2 \\ 1/3 & 0 & -1/5 \\ 1/2 & 1/5 & 0 \\ 3/5 & 1/3 & 1/7 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix}, \begin{bmatrix} 3 & 5/2 & 7/3 \\ 6 & 5 & 14/3 \\ 9 & 15/2 & 7 \\ 12 & 10 & 28/3 \end{bmatrix}$ **)**

Q8. Find x, y, z ,t if $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

(ANS: $x = 3, z = 9, y = 6 \text{ and } t = 6$**)**

Q9. Solve the matrix equation $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3\begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$ **.**

(ANS: $x = 1, 2$ **and** $y = 3 \pm 3\sqrt{2}$ **.)**

Q10. Find matrices X and Y, if $2X - Y = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$ **and** $X + 2Y = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$**.**

(ANS: $X = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}, Y = \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$**)**

Q11. Prove that the product of matrices $\begin{bmatrix} \cos^2\theta & \cos\theta \sin\theta \\ \cos\theta \sin\theta & \sin^2\theta \end{bmatrix}$ **and** $\begin{bmatrix} \cos^2\phi & \cos\phi \sin\phi \\ \cos\phi \sin\phi & \sin^2\phi \end{bmatrix}$ **is the null matrix when** θ and ϕ **differ by an odd multiple of** $\frac{\pi}{2}$**.**

Q12. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ **and** $(A + B)^2 = A^2 + B^2$**, find a and b.**

(ANS: $a = 1, b = 4$**)**

Q13. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$**, find x and y such that** $(xI + yA)^2 = A$**.**

(ANS: $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ or $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ **or** $\left(\frac{i}{\sqrt{2}}, -\frac{i}{\sqrt{2}}\right)$ or $\left(-\frac{i}{\sqrt{2}}, \frac{i}{\sqrt{2}}\right)$**)**

Q14. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$**,** $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$**,** $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$**. Find a matrix** D **such that** $CD - AB = 0$**.**

(ANS: $\begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$**)**

Q15. Find the value of 'x' such that:

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0.$$

Q16. If $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$**, find A.**

(ANS: $\begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$**)**

Q17. Let $f(x) = x^2 - 5x + 6$**.Find** $f(A)$ **if** $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$**.**

(ANS: $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$**)**

Q18. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ **and** $f(x) = x^2 - 4x + 7$**.Show that** $f(A) = O$**.Use this result to find** A^5**.**

(ANS: $A^5 = \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$**)**

Q19. Prove the following by the principle of mathematical induction:

If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$**, then** $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ **for every positive integer n.**

Q20. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$**, then prove that**

(i) $A_\alpha . A_\beta = A_{\alpha+\beta}$ **(ii)** $(A_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$**, for every positive integer n.**

Q21. If 'a' is a non-zero real or complex number. Use the principle of mathematical induction to prove that

If $A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}$**, then** $A^n = \begin{bmatrix} a^n & na^{n-1} \\ 0 & a^n \end{bmatrix}$ **for every positive integer n.**

Q22. Under what condition is the matrix equation $A^2 - B^2 = (A - B)(A + B)$ **is true?**

Q23. If $AB = A$ **and** $BA = B$**, then show that** $A^2 = A, B^2 = B$**.**

Q24.If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ **is a matrix satisfying** $AA^T = 9I_3$**, then find the values of 'a' and 'b'.**
(ANS: $a = -2$ **and** $b = -1$**)**

Q25. Find the values of x, y, z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ **satisfy the equation** $A^T A = I_3$**.**
(ANS: $x = \pm \frac{1}{\sqrt{2}}, y = \pm \frac{1}{\sqrt{6}}, z = \pm \frac{1}{\sqrt{3}}$**)**

ASSIGNMENT - DETERMINANTS

Show that each one of the following systems of equations is inconsistent.

1. $x + 2y = 9;$
   $2x + 4y = 7.$

2. $2x + 3y = 5;$
   $6x + 9y = 10.$

3. $4x - 2y = 3;$
   $6x - 3y = 5.$

4. $6x + 4y = 5;$
   $9x + 6y = 8.$

5. $x + y - 2z = 5;$
   $x - 2y + z = -2;$
   $-2x + y + z = 4.$

6. $2x - y + 3z = 1;$
   $3x - 2y + 5z = -4;$
   $5x - 4y + 9z = 14.$

7. $x + 2y + 4z = 12;$
   $y + 2z = -1;$
   $3x + 2y + 4z = 4.$

8. $3x - y - 2z = 2;$
   $2y - z = -1;$
   $3x - 5y = 3.$

Solve each of the following systems of equations using matrix method.

9. $5x + 2y = 4;$
   $7x + 3y = 5.$

10. $3x + 4y - 5 = 0;$
    $x - y + 3 = 0.$

11. $x + 2y = 1;$
    $3x + y = 4.$

12. $5x + 7y + 2 = 0;$
    $4x + 6y + 3 = 0.$

13. $2x - 3y + 1 = 0;$
    $x + 4y + 3 = 0.$

14. $4x - 3y = 3;$
    $3x - 5y = 7.$

15. $2x + 8y + 5z = 5;$
    $x + y + z = -2;$
    $x + 2y - z = 2.$ **[CBSE 2009C]**

16. $x - y + z = 1;$
    $2x + y - z = 2;$
    $x - 2y - z = 4.$ **[CBSE 2006C]**

17. $3x + 4y + 7z = 4;$
    $2x - y + 3z = -3;$
    $x + 2y - 3z = 8.$ **[CBSE 2012]**

18. $x + 2y + z = 7;$
    $x + 3z = 11;$
    $2x - 3y = 1.$ **[CBSE 2005, '08, '11]**

19. $2x - 3y + 5z = 16;$
    $3x + 2y - 4z = -4;$
    $x + y - 2z = -3.$ **[CBSE 2005C]**

20. $x + y + z = 4;$
    $2x - y + z = -1;$
    $2x + y - 3z = -9.$ **[CBSE 2005]**

21. $2x - 3y + 5z = 11;$
    $3x + 2y - 4z = -5;$
    $x + y - 2z = -3.$ **[CBSE 2009]**

22. $x + y + z = 1;$
    $x - 2y + 3z = 2;$
    $5x - 3y + z = 3.$ **[CBSE 2004, '09C]**

23. $x + y + z = 6;$
    $x + 2z = 7;$
    $3x + y + z = 12.$ **[CBSE 2009]**

24. $2x + 3y + 3z = 5;$
    $x - 2y + z = -4;$
    $3x - y - 2z = 3.$ **[CBSE 2008C, '12]**

25. $4x - 5y - 11z = 12;$
    $x - 3y + z = 1;$
    $2x + 3y - 7z = 2.$ **[CBSE 2007]**

26. $x - y + 2z = 7;$
    $3x + 4y - 5z = -5;$
    $2x - y + 3z = 12.$ **[CBSE 2012]**

27. $6x - 9y - 20z = -4;$
    $4x - 15y + 10z = -1;$
    $2x - 3y - 5z = -1.$

28. $3x - 4y + 2z = -1;$
    $2x + 3y + 5z = 7;$
    $x + z = 2.$ **[CBSE 2011C]**

29. $x + y - z = 1;$
    $3x + y - 2z = 3;$
    $x - y - z = -1.$ **[CBSE 2004]**

30. $2x + y - z = 1;$
    $x - y + z = 2;$
    $3x + y - 2z = -1.$ **[CBSE 2004C]**

31. $x + 2y + z = 4;$
$-x + y + z = 0;$
$x - 3y + z = 4.$ **[CBSE 2012C]**

32. $x - y - 2z = 3;$
$x + y = 1;$
$x + z = -6.$

33. $5x - y = -7;$
$2x + 3z = 1;$
$3y - z = 5.$

34. $x - 2y + z = 0;$
$y - z = 2;$
$2x - 3z = 10.$

35. $x - y = 3;$
$2x + 3y + 4z = 17;$
$y + 2z = 7.$ **[CBSE 2003C, '07C]**

36. $4x + 3y + 2z = 60;$
$x + 2y + 3z = 45;$
$6x + 2y + 3z = 70.$ **[CBSE 2011]**

37. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. **[CBSE 2007C, '08C]**

Using A^{-1}, solve the following system of equations:
$2x - 3y + 5z = 11;$
$3x + 2y - 4z = -5;$
$x + y - 2z = -3.$

38. If $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$, find A^{-1}.

Using A^{-1}, solve the following system of linear equations:
$2x + y + z = 1;$
$x - 2y - z = \frac{3}{2};$
$3y - 5z = 9.$

HINT: Here $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$.

39. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$, find AB.

Hence, solve the system of equations:
$x - 2y = 10$, $2x + y + 3z = 8$ and $-2y + z = 7.$ **[CBSE 2011]**

HINT: $AB = (11)\, I \Rightarrow A\left(\frac{1}{11}B\right) = I \Rightarrow A^{-1} = \left(\frac{1}{11}\right)B.$

Using matrices, solve the following system of equations.

40. $\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10,\ \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10,\ \frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$ **[CBSE 2007C]**

41. $\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4;\ \frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 0;\ \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2.\ (x, y, z \neq 0)$

ASSIGNMENT – XII – INVERSE TRIGONOMETRIC FUNCTIONS

Following questions carry 1 marks each

1. Evaluate: $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$.
2. Show that $\sin^{-1}\left(2x\sqrt{1-x^2}\right)=2\sin^{-1}x$
3. Write the domain of the function $\cos ec^{-1}x$.
4. Write the value of $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$.
5. Write the principal value of $\cos^{-1}\left(\frac{1}{2}\right)-2\sin^{-1}\left(-\frac{1}{2}\right)$.
6. Write $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right), |x|>1$ in simplest form.
7. Write the value of $\sin\left(2\sin^{-1}\frac{3}{5}\right)$ and $\tan\left(2\tan^{-1}\frac{1}{5}\right)$.
8. Write the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$.
9. If $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}$, $xy<1$, then write the value of $x+y+xy$.
10. If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then find the value of 'x'.

Following questions carry 4 marks each

11. Prove that: $2\tan^{-1}\left(\frac{1}{5}\right)+\sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)+2\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$.
12. Prove that: $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\ -\frac{1}{\sqrt{2}}\le x\le 1$.
13. Prove that: $\cot^{-1}7+\cot^{-1}8+\cot^{-1}18=\cot^{-1}3$.
14. Prove that: $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$.
15. Show that: $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right)\right]=\frac{4-\sqrt{7}}{3}$.
16. If $y=\cot^{-1}\left(\sqrt{\cos x}\right)-\tan^{-1}\left(\sqrt{\cos x}\right)$, then prove that $\sin y=\tan^2\left(\frac{x}{2}\right)$.

17. Prove that: $\tan^{-1}\left(\dfrac{\cos x}{1+\sin x}\right)=\dfrac{\pi}{4}-\dfrac{x}{2},\ x\in\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$.

18. Prove that: $\sin^{-1}\left(\dfrac{8}{17}\right)+\sin^{-1}\left(\dfrac{3}{5}\right)=\cos^{-1}\left(\dfrac{36}{85}\right)$.

19. Prove that: $\cos\left(\sin^{-1}\dfrac{3}{5}+\cot^{-1}\dfrac{3}{2}\right)=\dfrac{6}{5\sqrt{13}}$.

20. Prove that: $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi$.

21. Prove that: $\cos\left[\tan^{-1}\left\{\sin\left(\cot^{-1}x\right)\right\}\right]=\sqrt{\dfrac{1+x^2}{2+x^2}}$.

22. Prove that: $\tan\left[\dfrac{\pi}{4}+\dfrac{1}{2}\cos^{-1}\left(\dfrac{a}{b}\right)\right]+\tan\left[\dfrac{\pi}{4}-\dfrac{1}{2}\cos^{-1}\left(\dfrac{a}{b}\right)\right]=\dfrac{2b}{a}$.

23. Prove that: $\sin^{-1}\left(\dfrac{4}{5}\right)+\sin^{-1}\left(\dfrac{5}{13}\right)+\sin^{-1}\left(\dfrac{16}{65}\right)=\dfrac{\pi}{2}$.

24. Solve: $\tan^{-1}\left(\dfrac{1-x}{1+x}\right)=\dfrac{1}{2}\tan^{-1}x\ ;x>0$. (Ans: $\dfrac{1}{\sqrt{3}}$)

25. Solve : $\tan^{-1}\left(x+1\right)+\tan^{-1}\left(x-1\right)=\tan^{-1}\left(\dfrac{8}{31}\right)$. (Ans: $\dfrac{1}{4}$)

26. Solve: $\cos^{-1}\left(\dfrac{x^2-1}{x^2+1}\right)+\tan^{-1}\left(\dfrac{2x}{x^2-1}\right)=\dfrac{2\pi}{3}$. (Ans: $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$)

27. Solve: $2\tan^{-1}\left(\sin x\right)=\tan^{-1}\left(2\sec x\right)$, where $x\neq\dfrac{\pi}{2}$ (Ans: $\dfrac{\pi}{4}$)

28. Solve: $\cos\left(\tan^{-1}x\right)=\sin\left(\cot^{-1}\dfrac{3}{4}\right)$. (Ans: $\pm\dfrac{3}{4}$)

29. Solve: $\cot^{-1}x-\cot^{-1}\left(x+2\right)=\dfrac{\pi}{12}$ (Ans: $\sqrt{3}$)

30. Solve: $\cos\left[2\sin^{-1}\left(-x\right)\right]=0$ (Ans: $x=\pm\dfrac{1}{\sqrt{2}}$)

31. Solve: $\sin^{-1}\left(1-x\right)+\sin^{-1}x=\cos^{-1}x$ (Ans: $x=0,\dfrac{1}{2}$)

32. Solve: $\sin^{-1}\left(6\sqrt{3}x\right)+\sin^{-1}\left(6x\right)=\dfrac{\pi}{2}$ (Ans: $x=\pm\dfrac{1}{12}$)

33. Solve: $\sin^{-1}\dfrac{15}{x}+\sin^{-1}\dfrac{8}{x}=\dfrac{\pi}{2}$. (Ans: $x=\pm17$)

34. Solve: $\tan^{-1}\left(\dfrac{x+1}{x-1}\right)+\tan^{-1}\left(\dfrac{x-1}{x}\right)=-\tan^{-1}7$ (Ans: 2)

Linear Programming Problem

1. If a young man drives his scooter at a speed of 25 km/hr, he has to spend ₹2 per km on petrol. If he drives the scooter at a speed of 40 km/hour, it produces air pollution and increases his expenditure on petrol to ₹5 per km. He has a maximum of ₹100 to spend on petrol and travel a maximum distance in one hour time with less pollution. Express this problem as an LPP and solve it graphically. What value do you find here?
   [CBSE 2013]
2. A manufacturer has three machines installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas Machine III must operate at least for 5 hours a day. He produces only two items, each requiring the use of the three machines. The number of hours required for producing one unit each of the items on the three machines is given in the following table:

Item	Machine I	Machine II	Machine III
A	1	2	1
B	2	1	5/4

   He makes a profit of ₹6.00 on item A and ₹4.00 on item B. Assuming that he can sell all that he produces, how many of each item should he produce so as to maximize his profit? Determine the maximum profit. Formulate the LPP mathematically and then solve it.

3. Two tailors, A and B earn ₹15 and ₹20 per day respectively. A can stitch 6 shirts and 4 pants while B can stitch 10 shirts and 4 pants per day. How many days shall each work if it is desired to produce at least 60 shirts and 32 pants at a minimum labour cost?
4. A factory manufactures two types of screws, A and B, each type requiring the use of two machines — an automatic and a hand-operated one. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machines to manufacture a package of screws 'A', while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a package of screws 'B'. Each machine is available for at most 4 hours on any day. The manufacturer can sell all the screws he can manufacture, how many packages of each type should the factory owner produce in a day in order to maximize his profit? Determine the maximum profit.
   Profit on package A = 70 paise
   Profit on package B = ₹1
   [CBSE 2018, NCERT]
5. A company produces two types of leather belts, say type A and B. Belt A is of superior quality and belt B is of lower quality. Profits on each type of belt are ₹2 and ₹1.50 per belt respectively. Each belt of type A requires twice as much time as required by a belt of type B. If all belts were of type B, the company could produce 1000 belts per day. But the supply of leather is sufficient only for 800 belts per day (both A and B combined). Belt A requires a fancy buckle and only 400 fancy buckles are available per day. For belt of type B, only 700 buckles are available per day.

How should the company manufacture the two types of belts in order to have a maximum overall profit?

6. A small manufacturer has employed 5 skilled men and 10 semi-skilled men and makes an article in two qualities — deluxe model and an ordinary model. The making of a deluxe model requires 2 hrs work by a skilled man and 3 hrs by a semi-skilled man. The ordinary model requires 1 hr by a skilled man and 1.5 hrs by a semi-skilled man. By union rules no man can work more than 8 hrs per day. The profit on deluxe model is ₹15 and on ordinary model is ₹10. How many articles of each type should be made in order to maximize his total daily profit?
[CBSE 2019]
7. A manufacturer makes two types A and B of tea-cups. Three machines are needed for the manufacture and the time in minutes required for each cup on the machines is given below:

Machines	**I**	**II**	**III**
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each cup A is 75 paise and that on each cup B is 50 paise, show that 15 tea-cups of type A and 30 of type B should be manufactured to get the maximum profit.
[CBSE 2003, 2008]
8. A factory owner purchases two types of machines, A and B, for his factory. The requirements and limitations for the machines are as follows:

	Area occupied	**Labour force required**	**Daily output**
Machine A	1000 sq. m	12 men	60 units
Machine B	1200 sq. m	8 men	40 units

He has an area of 7000 sq. m available and 72 skilled men who can operate the machines. How many machines of each type should he buy to maximize the daily output?
9. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 gm of silver and 1 gm of gold while that of type B requires 1 gm of silver and 2 gm of gold. The company can produce 9 gm of silver and 8 gm of gold. If each unit of type A brings a profit of ₹120 and type B ₹150, find the number of units of each type that the company should produce to maximize profit. What is the maximum profit?
[CBSE 2020]
10. A manufacturer of Furniture makes two products: chairs and tables. Processing of these products is done on two machines A and B. A chair requires 2 hrs on machine A and 6 hrs on machine B. A table requires 4 hrs on machine A and 2 hrs on machine B. There are 16 hrs time per day available on machine A and 30 hrs

on machine B. Profit gained by the manufacturer from a chair and a table is ₹3 and ₹5 respectively. Find with the help of graph what should be the daily production of each of the two products so as to maximize his profit.

11. A furniture manufacturing company plans to make two products: chairs and tables. From its available resources which consists of 400 square feet of wood and 450 man-hours, it is known that to make a chair requires 5 square feet of wood and 10 man-hours and yields a profit of ₹45, while each table uses 20 square feet of wood and 25 man-hours and yields a profit of ₹80. How many items of each product should be produced by the company so that the profit is maximum?
12. A firm manufactures two products A and B. Each product is processed on two machines M_1 and M_2. Product A requires 4 minutes of processing time on M_1 and 8 minutes on M_2; product B requires 4 minutes on M_1 and 4 minutes on M_2. Machine M_1 is available for not more than 8 hrs 20 mins while machine M_2 is available for 10 hrs during any working day. The products A and B are sold at a profit of ₹3 and ₹4 respectively. Formulate the problem as a linear programming problem and find how many products of each type should be produced by the firm to get maximum profit.
13. A firm wants to produce two types of electric items, A and B, and can make a profit of ₹20 per unit of A and ₹30 per unit of B. Each unit of A requires 3 motors and 4 transformers, and each unit of B requires 2 motors and 4 transformers. The total supply of these per month is restricted to 210 motors and 300 transformers. Type B is an export model requiring a voltage stabilizer which has a supply restricted to 65 units per month. Formulate the linear programming problem for maximum profit and solve it graphically.
14. A factory uses three different resources for the manufacture of two different products, 20 units of resources A, 12 units of B and 16 units of C being available. 1 unit of the first product requires 2, 2 and 4 units of the respective resources and 1 unit of the second product requires 4, 2 and 0 units of the respective resources. It is known that the first product gives a profit of 2 monetary units per unit and the second 3. Formulate the LPP. How many units of each product should be manufactured for maximizing the profit? Solve it graphically.
15. A publisher sells a hard cover edition of a textbook for ₹72.00 and a paperback edition of the same text for ₹40.00. Costs to the publisher are ₹56.00 and ₹28.00 per book respectively in addition to weekly costs of ₹9600.00. Both types require 5 minutes of printing time, although hardcover requires 10 minutes binding time and the paperback requires only 2 minutes. Both the printing and binding operations have 4,800 minutes available each week. How many of each type of book should be produced in order to maximize profit?

16. A firm manufactures headache pills in two sizes A and B. Size A contains 2 grains of aspirin, 5 grains of bicarbonate and 1 grain of codeine; size B contains 1 grain of aspirin, 8 grains of bicarbonate and 6 grains of codeine. It has been found by users that it requires at least 12 grains of aspirin, 74 grains of bicarbonate and 24 grains of

codeine for providing immediate relief. Determine graphically the least number of pills a patient should have to get immediate relief. Determine also the quantity of codeine consumed by the patient.

17. A chemical company produces two compounds, A and B. The following table gives the units of ingredients C and D per kg of compounds A and B as well as minimum requirements of C and D and costs per kg of A and B. Find the quantities of A and B which will give a supply of C and D at a minimum cost.

Compound / Ingredient	**A**	**B**	**Minimum Requirement**
Ingredient C	1	2	80
Ingredient D	3	1	75

Cost (in ₹) per kg:
A = ₹4, B = ₹6

18. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours available for assembling. The profit is 50 paise each for type A and 60 paise each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize the profit?
[NCERT, CBSE 2020]

19. A manufacturer makes two products A and B. Product A sells at ₹200 each and takes $\frac{1}{2}$ hour to make. Product B sells at ₹300 each and takes 1 hour to make. There is a permanent order for 14 units of product A and 16 units of product B. A working week consists of 40 hours of production and weekly turnover must not be less than ₹10000. If the profit on each product A is ₹20 and on product B is ₹30, then how many of each product should be produced so that the profit is maximum. Also, find the maximum profit.

20. A manufacturer produces two types of steel trunks. He has two machines A and B for completing them, the first type of trunk requires 3 hours on machine A and 3 hours on machine B, whereas the second type of trunk requires 3 hours on machine A and 2 hours on machine B. Machines A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of ₹30 and ₹25 per trunk of the first type and second type respectively. How many trunks of each type must he make each day to make the maximum profit?
[CBSE 2001, 2005, 2012]

21. A manufacturer of patent medicines is preparing a production plan for two categories of medicines, A and B. There are sufficient raw materials available to make 2000 bottles of A and 4000 bottles of B, but there are only 4500 bottles into which either of the medicines can be put. Further, it takes 3 hours to prepare enough material to fill 1000 bottles of A, while it takes 1 hour to prepare enough material to fill 1000 bottles of B and there are 6 hours available for this operation. The profit is ₹8 per bottle for A and ₹7 per bottle for B. How should the manufacturer schedule his production in order to maximize his profit?

22. An aeroplane can carry a maximum of 200 passengers. A profit of ₹400 is made on each first class ticket and a profit of ₹600 is made on each economy class ticket. The airline reserves at least 20 seats for first class. However, at least 4 times as many passengers prefer to travel by economy class than first class. Determine how many of each type of tickets must be sold in order to maximize the profit for the airline. What is the maximum profit?
23. A gardener has supply of two types of fertilizers F_1 and F_2 which consists of 10% nitrogen and 6% phosphoric acid, and type II fertilizer which consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, he finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crops. If type I fertilizer costs 60 paise per kg and type II fertilizer costs 40 paise per kg, determine how many kilograms of each fertilizer should be used so that nutrient requirements are met at minimum cost. What is the minimum cost?
[CBSE 2002, 2005]
24. Anil wants to invest at most ₹1200 in Saving Certificates and National Saving Bonds. According to rules, he has to invest at least ₹200 in Saving Certificates and at least ₹400 in National Saving Bonds. If the rate of interest on Saving Certificates is 8% per annum and the rate of interest on National Saving Bonds is 10% per annum, how much money should he invest to earn maximum yearly income? Also find his maximum yearly income.
25. A man owns a field of area 1000 sq.m. He wants to plant fruit trees in it. He has a sum of ₹1400 to purchase young trees. He has the choice of two types of trees. Type A requires 10 sq.m of ground per tree and costs ₹20 per tree and type B requires 20 sq.m of ground per tree and costs ₹25 per tree. When fully grown, type A produces an average of 20 kg of fruit which can be sold at a profit of ₹2.00 per kg and type B produces an average of 40 kg of fruit which can be sold at a profit of ₹1.50 per kg. How many of each type should be planted to achieve maximum profit when the trees are fully grown? What is the maximum profit?
26. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of grinding/cutting machine and a sprayer. It takes 2 hours on the grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp while it takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at most 20 hours and the grinding/cutting machine for at most 12 hours. The profit from the sale of a lamp is ₹5.00 and a shade is ₹3.00. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximize his profit?
[NCERT, CBSE 2013]
27. A producer has 30 and 17 units of labour and capital respectively which he can use to produce two types of goods X and Y. To produce one unit of X, 2 units of labour and 3 units of capital are required. Similarly, 3 units of labour and 1 unit of

capital is required to produce one unit of Y. If X and Y are priced at ₹100 and ₹120 per unit respectively, how should the producer use his resources to maximize the total revenue? Solve the problem graphically.

[CBSE 2001]

28. A firm manufactures two types of products A and B and sells them at a profit of ₹5 per unit of A and ₹3 per unit of B. Each product is processed on two machines M_1 and M_2. One unit of type A requires one minute of processing time on M_1 and two minutes of processing on M_2, whereas one unit of type B requires one minute of processing on M_1 and one minute on M_2. Machines M_1 and M_2 are respectively available for at most 5 hours and 6 hours in a day. Find how many units of each product should the firm produce a day in order to maximize the profit. Solve the problem graphically.

    [CBSE 2010]

29. A small firm manufactures items A and B. The total number of items of A and B that it can manufacture in a day is at most 24. Item A takes one hour to make while item B takes only half an hour. The maximum time available per day is 16 hours. If the profit on one unit of item A be ₹300 and one unit of item B be ₹160, how many of each type of item should be produced to maximize the profit? Solve the problem graphically.

    [CBSE 2001, 2004]

30. A company manufactures two types of toys A and B. Type A requires 5 minutes each for cutting and 10 minutes each for assembling. Type B requires 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours available for cutting and 4 hours available for assembling in a day. The profit is 50 paise on type A and 60 paise on type B. How many toys of each type should the company manufacture in a day to maximize the profit?

    [CBSE 2001]

31. A company manufactures two articles A and B. There are two departments through which these articles are processed:
    (i) assembly and
    (ii) finishing departments.

    The maximum capacity of the first department is 60 hours a week and that of the other department is 48 hours per week. The production of each unit of article A requires 4 hours in assembly and 2 hours in finishing and each unit of article B requires 2 hours in assembly and 4 hours in finishing. If the profit is ₹6 for each unit of A and ₹8 for each unit of B, find the number of units of A and B to be produced per week to maximize the profit.

    [CBSE 2003]

32. A firm makes items A and B and the total number of items it can make in a day is 24. It takes one hour to make an item of A and half an hour to make an item of B. The maximum time available per day is 16 hours. The profit on an item of A is ₹300 and

on one item of B is ₹160. Formulate the problem mathematically and solve it graphically.
[CBSE 2004]

33. A company sells two different products, A and B. The two products are produced in a common production process, which has a total capacity of 500 man-hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The market has been surveyed and the company officials feel that the maximum number of units of A that can be sold is 70 and for B is 125. If the profit is ₹20 per unit for the product A and ₹15 per unit for the product B, how many units of each product should be sold to maximize the profit?
34. A box manufacturer makes large and small boxes from a large piece of cardboard. The large boxes require 4 sq. metre per box while the small boxes require 3 sq. metre per box. The manufacturer is required to make at least three large boxes and at least twice as many small boxes as large boxes. If 60 sq. metre of cardboard is in stock, and the profits on the large and small boxes are ₹3 and ₹2 per box, how many of each box should be made to maximize the total profit?
35. A manufacturer makes two products, A and B. Product A sells at ₹200 each and takes $\frac{1}{2}$hour to make. Product B sells at ₹300 each and takes 1 hour to make. There is a permanent order for 14 units of product A and 16 units of product B. A working week consists of 40 hours of production and the weekly turnover must not be less than ₹10000. If the profit on each product A is ₹20 and on product B is ₹30, then how many of each product should be produced so that the profit is maximum? Also find the maximum profit.
36. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹300 and that on a chain is ₹190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an LPP and solve it graphically.
[CBSE 2010]

37. A library has to accommodate two different types of books on a shelf. The books are 6 cm and 4 cm thick and weigh 1 kg and $1\frac{1}{2}$kg each respectively. The shelf is 96 cm long and can support at most 21 kg. How should the shelf be filled with the books of two types in order to include the greatest number of books? Make it as an LPP and solve it graphically.
[CBSE 2010]