# ASSIGNMENT
# CH-INVERSE TRIGONOMETRIC FUNCTIONS

## Q1. Evaluate the following:

**(i)** $\sin\left(2\sin^{-1}0.6\right)$ **(ii)** $\sin\left(3\sin^{-1}0.4\right)$ **(iii)** $\tan\left(2\tan^{-1}\frac{1}{5}\right)$ **(iv)** $\sin^{-1}\left(\sin\frac{5\pi}{6}\right)$

**(v)** $\cos^{-1}\left\{\cos\left(-\frac{\pi}{4}\right)\right\}$ **(vi)** $\sin\left\{\frac{\pi}{3}-\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right\}$ **(vii)** $\cos\left\{\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\frac{\pi}{4}\right\}$

**(viii)** $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

**ANS: (i)** $0.96$ **(ii)** $0.944$ **(iii)** $\frac{5}{12}$ **(iv)** $\frac{\pi}{6}$ **(v)** $\frac{\pi}{4}$ **(vi)** $\frac{\sqrt{3}}{2}$ **(vii)** $-\frac{\sqrt{3}+1}{2\sqrt{2}}$

## Q2. Express the following in the simplest form:

**(i)** $\tan^{-1}\left\{\sqrt{\frac{1-\cos x}{1+\cos x}}\right\}, 0<x<\pi$ **(ANS:** $\frac{x}{2}$**)**

**(ii)** $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right), -\frac{\pi}{2}<x<\frac{\pi}{2}$ **(ANS:** $\frac{\pi}{4}-\frac{x}{2}$**)**

**(iii)** $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{\pi}{2}<x<\frac{\pi}{2}$ **(ANS:** $\frac{\pi}{4}+\frac{x}{2}$**)**

**(iv)** $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right), -\frac{\pi}{4}<x<\frac{\pi}{4}$ **(ANS:** $\frac{\pi}{4}-x$**)**

## Q3. Write the following in the simplest form:

**(i)** $\tan^{-1}\left\{\frac{x}{\sqrt{a^2-x^2}}\right\}, -a<x<a$ **(ii)** $\tan^{-1}\left\{\sqrt{\frac{a-x}{a+x}}\right\}, -a<x<a$

**(iii)** $\sin^{-1}\left\{\frac{x}{\sqrt{x^2+a^2}}\right\}$ **(iv)** $\cos^{-1}\left\{\frac{x}{\sqrt{x^2+a^2}}\right\}$

**(ANS: (i)** $\sin^{-1}\frac{x}{a}$ **(ii)** $\frac{1}{2}\cos^{-1}\frac{x}{a}$ **(iii)** $\tan^{-1}\frac{x}{a}$ **(iv)** $\cot^{-1}\frac{x}{a}$**)**

## Q4. Prove that:

**(i)** $\sin\left(\cot^{-1}x\right)=\frac{1}{\sqrt{1+x^2}}$ **(ii)** $\cos\left(\tan^{-1}x\right)=\frac{1}{\sqrt{1+x^2}}$**.**

## Q5. Prove the following:

**(i)** $\sin^{-1}\frac{12}{12}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$ **(ii)** $\tan^{-1}2+\tan^{-1}3=\frac{3\pi}{4}$

**(iii)** $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$

**(iv)** $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]=\frac{x+y}{1-xy}$**, if** $|x|<1, y>0$ and $xy>1$

**(v)** $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0,1]$

## Q6. Prove the following:

**(i)** $\tan^{-1}\left\{\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right\} = \frac{\pi}{4}+\frac{x}{2}, 0 < x < \frac{\pi}{2}$

**(ii)** $\cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\} = \frac{x}{2}, 0 < x < \frac{\pi}{2}$

**(iii)** $\tan^{-1}\left\{\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right\} = \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x, 0 < x < 1$

**(iv)** $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\} = \frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2, -1 < x < 1$

**(v)** $\tan^{-1}x+\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), |x| < \frac{1}{3}$

**(vi)** $\cos^{-1}x = 2\sin^{-1}\sqrt{\frac{1-x}{2}} = 2\cos^{-1}\sqrt{\frac{1+x}{2}}$

**(vii)** $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0,1]$

## Simplify:

**(v)** $\cos^{-1}\left(\frac{3}{5}\cos x+\frac{4}{5}\sin x\right)$ **(vi)** $\sin^{-1}\left(\frac{5}{13}\cos x+\frac{12}{13}\sin x\right)$

**(ANS: (v)** $x-\tan^{-1}\frac{4}{3}$ **(vi)** $x+\tan^{-1}\frac{5}{12}$ **)**

**(vii)** $\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right), -\frac{\pi}{4} < x < \frac{\pi}{4}$ **(viii)** $\cos^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right), \frac{\pi}{4} < x < \frac{5\pi}{4}$

**(ANS: (vii)** $x+\frac{\pi}{4}$ **(viii)** $x-\frac{\pi}{4}$ **)**

## Q7. Prove that:

**(i)** $\sec^2\left(\tan^{-1}2\right)+\operatorname{cosec}^2\left(\cot^{-1}3\right) = 15$ **(ii)** $\sin\left[\cot^{-1}\left\{\cos\left(\tan^{-1}x\right)\right\}\right] = \sqrt{\frac{x^2+1}{x^2+2}}$

**(iii)** $\cos\left[\tan^{-1}\left\{\sin\left(\cot^{-1}x\right)\right\}\right] = \sqrt{\frac{x^2+1}{x^2+2}}$ **(iv)** $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

**(v)** $\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{3}{5}-\tan^{-1}\frac{8}{19} = \frac{\pi}{4}$ **(vi)** $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

**(vii)** $\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{65} = \frac{\pi}{2}$ **(viii)** $\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

**(ix)** $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{70}+\tan^{-1}\frac{1}{99}=\frac{\pi}{4}$ **(x)** $2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{5\sqrt{2}}{7}+2\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

**Q8. If** $a>b>c>0$**, prove that**

$$\cot^{-1}\left(\frac{ab+1}{a-b}\right)+\cot^{-1}\left(\frac{bc+1}{b-c}\right)+\cot^{-1}\left(\frac{ca+1}{c-a}\right)=\pi$$

**Q9. If** $y=\cot^{-1}\left(\sqrt{\cos x}\right)-\tan^{-1}\left(\sqrt{\cos x}\right)$**, prove that** $\sin y=\tan^2\frac{x}{2}$

**Q10. Prove that** $\tan^{-1}\frac{yz}{xr}+\tan^{-1}\frac{zx}{yr}+\tan^{-1}\frac{xy}{zr}=\frac{\pi}{2}$**, where** $x^2+y^2+z^2=r^2$

**Q11. If** $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha$ **, prove that** $\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha$

**Q12. If** $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ **, prove that** $x^2+y^2+z^2+2xyz=1$**.**

**Q13 If** $\sin^{-1}x+\sin^{-1}y+\sin^{-1}$**, prove that**

**(i)** $x\sqrt{1-x^2}+y\sqrt{1-y^2}+z\sqrt{1-z^2}=2xyz$

**(ii)** $x^4+y^4+z^4+4x^2y^2z^2=2\left(x^2y^2+y^2z^2+z^2x^2\right)$

**Q14. If** $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}=\alpha$ **, then prove that** $x^2=\sin 2\alpha$

**Q15. Prove that** $\cos^{-1}\left(\frac{\cos\alpha+\cos\beta}{1+\cos\alpha\ \cos}\right)=2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right)$**.**

**Q16. Evaluate** $\cos\left(2\cos^{-1}x+\sin^{-1}x\right)$ **at** $x=\frac{1}{5}$**,** $0\le\cos^{-1}x\le\pi$ **and** $-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}$**. (ANS:** $-\sqrt{\frac{24}{25}}$ **)**

**Q17. Prove that:**

$$\frac{\alpha^3}{2}\operatorname{cosec}^2\left(\frac{1}{2}\tan^{-1}\frac{\alpha}{\beta}\right)+\frac{\beta^2}{2}\sec^2\left(\frac{1}{2}\tan^{-1}\frac{\beta}{\alpha}\right)=(\alpha+\beta)\left(\alpha^2+\beta^2\right)$$

**Q18. prove that:**

$$\tan^{-1}\frac{1-x}{1+x}-\tan^{-1}\frac{1-y}{1+y}=\sin^{-1}\frac{y-x}{\sqrt{1+x^2}\sqrt{1+y^2}}$$

**Q19. Prove that:**

$$\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}=\frac{2b}{a}$$

**Q20. Prove that:**

$$\tan^{-1}\left\{\frac{\cos 2\alpha\ \sec 2\beta+\cos 2\beta\ \sec 2\alpha}{2}\right\}=\tan^{-1}\left\{\tan^2(\alpha+\beta)\tan^2(\alpha-\beta)\right\}+\tan^{-1}1$$

**Q21. If** $x = \operatorname{cosec}\left[\tan^{-1}\left\{\cos\left(\cot^{-1}\left(\sec\left(\sin^{-1} a\right)\right)\right)\right\}\right]$ **and** $y = \sec\left[\cot^{-1}\left\{\sin\left(\tan^{-1}\left(\operatorname{cosec}\left(\cos^{-1} a\right)\right)\right)\right\}\right]$**, where** $a \in [0,1]$ **.Find the relationship between *x* and *y* in terms of '*a*'.**

**Q22. Simplify each of the following:**

**(i)** $\tan^{-1}\left(\frac{a+bx}{b-ax}\right), x < \frac{b}{a}$ **(ANS:** $\tan^{-1}\frac{a}{b} + \tan^{-1} x$**)**

**(ii)** $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right), -\frac{\pi}{2} < x < \frac{\pi}{2}, \frac{a}{b}\tan x > -1$ **(ANS:** $\tan^{-1}\frac{a}{b} - x$**)**

**(iii)** $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right), -\frac{1}{\sqrt{3}} < \frac{x}{a} < \frac{1}{\sqrt{3}}$ **(ANS:** $3\tan^{-1}\frac{x}{a}$**)**

**(iv)** $\sin^{-1}\left\{x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right\}$ **(ANS:** $\sin^{-1} x - \sin^{-1}\sqrt{x}$**)**

**(v)** $\tan^{-1}\left\{x + \sqrt{1+x^2}\right\} x \in R$ **(ANS:** $\frac{\pi}{2} - \frac{1}{2}\cot^{-1} x$**)**

**(vi)** $\tan^{-1}\left\{\sqrt{1+x^2} - x\right\}, x \in R$ **(ANS:** $\frac{1}{2}\cot^{-1} x$**)**

**(vii)** $\tan^{-1}\left\{\frac{\sqrt{1+x^2} - 1}{x}\right\}, x \neq 0$ **(ANS:** $\frac{1}{2}\tan^{-1} x$**)**

**(viii)** $\tan^{-1}\left\{\frac{\sqrt{1+x^2} + 1}{x}\right\}, x \neq 0$ **(ANS:** $\frac{\pi}{2} - \frac{1}{2}\tan^{-1} x$**)**

**(ix)** $\tan^{-1}\sqrt{\frac{a-x}{a+x}}, -a < x < a$ **(ANS:** $\frac{1}{2}\cos^{-1}\frac{x}{a}$**)**

**(x)** $\tan^{-1}\left\{\frac{x}{a + \sqrt{a^2 - x^2}}\right\}, -a < x < a$ **(ANS:** $\frac{1}{2}\sin^{-1}\frac{x}{a}$**)**

**(xi)** $\sin^{-1}\left\{\frac{x + \sqrt{1-x^2}}{\sqrt{2}}\right\}, -\frac{\pi}{4} < x < \frac{\pi}{4}$ **(ANS:** $\frac{\pi}{4} + \sin^{-1} x$**)**

**(xii)** $\sin^{-1}\left\{\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{2}}\right\}, 0 < x < 1$ **(ANS:** $\frac{\pi}{4} + \frac{1}{2}\cos^{-1} x$**)**

**(xiii)** $\sin\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$ **(ANS:** $\sqrt{1-x^2}$ **)**

**(xiv)** $\cot^{-1}\frac{a}{\sqrt{x^2 - a^2}}, |x| > a$ **(ANS:** $\sec^{-1}\frac{x}{a}$**)**

**Q21. Solve the following equations for 'x':**

**(i)** $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$ **(ii)** $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

**(iii)** $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$ **(iv)** $\tan^{-1}\frac{x-1}{x+1}+\tan^{-1}\frac{2x-1}{2x+1}=\tan^{-1}\frac{23}{36}$

**(v)** $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec}x)$ **(vi)** $\sin^{-1}\frac{3x}{5}+\sin^{-1}\frac{4x}{5}=\sin^{-1}x$

**(vii)** $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$ **(viii)** $\sin\left[2\cos^{-1}\left\{\cot\left(2\tan^{-1}x\right)\right\}\right]=0$

**(ix)** $\tan^{-1}\sqrt{x^2+x}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ **(x)** $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

**(xi)** $\tan^{-1}\frac{1}{4}+2\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{6}+\tan^{-1}\frac{1}{x}=\frac{\pi}{4}$ **(xii)** $\sin^{-1}x+\sin^{-1}2x=\frac{\pi}{3}$

**(xiii)** $3\sin^{-1}\frac{2x}{1+x^2}-4\cos^{-1}\frac{1-x^2}{1+x^2}+2\tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{3}$

**(xiv)** $\cos^{-1}x+\sin^{-1}\frac{x}{2}=\frac{\pi}{2}$ **(xv)** $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$

**(xvi)** $\tan\left(\cos^{-1}x\right)=\sin\left(\cot^{-1}\frac{1}{2}\right)$ **(xvii)** $\tan^{-1}\left(\frac{1-x}{1+x}\right)-\frac{1}{2}\tan^{-1}x=0$, **where** $x>0$

**(xviii)** $\cot^{-1}x-\cot^{-1}(x+2)=\frac{\pi}{12}$, **where** $x>0$

**(ANS: (i)** $x=\frac{1}{5}$, **(ii)** $x=\pm\frac{1}{\sqrt{2}}$, **(iii)** $x=\frac{1}{6}$, **(iv)** $x=\frac{4}{3}$, **(v)** $x=\frac{\pi}{4}$, **(vi)** $x=\pm1$, **(vii)** $x=0$, **(viii)** $x=\pm1,-1\pm\sqrt{2},1\pm\sqrt{2}$, **(ix)** $x=0,-1$ **(x)** $x=\frac{1}{4}$, **(xi)** $x=-\frac{461}{9}$,
**(xii)** $x=\frac{1}{2}\sqrt{\frac{3}{7}}$, **(xiii)** $x=\frac{1}{\sqrt{3}}$ **(xiv)** $x=1$, **(xv)** $x=0,\pm\frac{1}{\sqrt{2}}$, **(xvi)** $x=\pm\frac{\sqrt{5}}{3}$,
**(xvii)** $x=\frac{1}{\sqrt{3}}$, **(xviii)** $x=\sqrt{3}$ **)**

# ASSIGNMENT
# CH-MATRICES

**Q1. If** $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$**, find *x,y,z,w*.**

**(ANS:** $x=1, y=2, z=3, w=4$**)**

**Q2. Find the value of *x, y ,a,* and *b* if**

$$\begin{bmatrix} 2x-3y & a-b & 3 \\ 1 & x+4y & 3a+4b \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$$

**(ANS:** $x=2, y=1, a=3, b=5$**)**

**Q3. For what values of x and y are the following matrices equal?**

$$A = \begin{bmatrix} 2x+1 & 2y \\ 0 & y^2-5y \end{bmatrix}, \quad B = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

**Q4. If** $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c-2 \\ 2b+4 & -21 & 0 \end{bmatrix}$**.Obtain the values of *a, b, c, x, y,* and *z*.**

**(ANS:** $a=-2, b=-7, c=-1, x=-3, y=-5, z=2$**)**

**Q5. Give an example of**

**(i) a row matrix which is also a column matrix,**

**(ii) a diagonal matrix which is not scalar,**

**(iii) a triangular matrix**

**Q6. Construct a** $2\times 3$ **matrix whose elements** $a_{ij}$ **are given by**

**(i)** $a_{ij} = \frac{(i+j)^2}{2}$ **(ii)** $a_{ij} = \frac{(i-j)^2}{2}$ **(iii)** $a_{ij} = \frac{(i-2j)^2}{2}$ **(iv)** $\frac{|2i-3j|}{2}$

**(v)** $a_{ij} = \frac{|-3i+j|}{2}$

**Q7. Construct a** $4\times 3$ **matrix whose elements** $a_{ij}$ **are given by**

**(i)** $a_{ij} = \frac{i-j}{i+j}$ **(ii)** $a_{ij} = i$ **(iii)** $a_{ij} = 2i + \frac{i}{j}$

**(ANS:** $\begin{bmatrix} 0 & -1/3 & -1/2 \\ 1/3 & 0 & -1/5 \\ 1/2 & 1/5 & 0 \\ 3/5 & 1/3 & 1/7 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix}$, $\begin{bmatrix} 3 & 5/2 & 7/3 \\ 6 & 5 & 14/3 \\ 9 & 15/2 & 7 \\ 12 & 10 & 28/3 \end{bmatrix}$**)**

**Q8. Find x, y, z ,t if** $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

**(ANS:** $x = 3, z = 9, y = 6 \text{ and } t = 6$**)**

**Q9. Solve the matrix equation** $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3\begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$ **.**

**(ANS:** $x = 1, 2$ **and** $y = 3 \pm 3\sqrt{2}$ **.)**

**Q10. Find matrices X and Y, if** $2X - Y = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$ **and**

$X + 2Y = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$ **.**

**(ANS:** $X = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}, Y = \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$**)**

**Q11. Prove that the product of matrices**

$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ **and** $\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ **is the null matrix when**

$\theta$ and $\phi$ **differ by an odd multiple of** $\frac{\pi}{2}$**.**

**Q12. If** $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ **and** $(A + B)^2 = A^2 + B^2$**, find a and b.**

**(ANS:** $a = 1, b = 4$**)**

**Q13. If** $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$**, find x and y such that** $(xI + yA)^2 = A$**.**

**(ANS:** $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ or $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ **or** $\left(\frac{i}{\sqrt{2}}, -\frac{i}{\sqrt{2}}\right)$ or $\left(-\frac{i}{\sqrt{2}}, \frac{i}{\sqrt{2}}\right)$**)**

**Q14. Let** $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$**,** $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$**,** $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$**. Find a matrix** $D$ **such that** $CD - AB = 0$**.**

**(ANS:** $\begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$**)**

**Q15. Find the value of 'x' such that:**

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0.$$

**Q16. If** $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$**, find A.**

**(ANS:** $\begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$**)**

**Q17. Let** $f(x) = x^2 - 5x + 6$**.Find** $f(A)$ **if** $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$**.**

**(ANS:** $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$**)**

**Q18. Let** $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ **and** $f(x) = x^2 - 4x + 7$**.Show that** $f(A) = O$**.Use this result to find** $A^5$**.**

**(ANS:** $A^5 = \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$**)**

**Q19. Prove the following by the principle of mathematical induction:**

**If** $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$**, then** $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ **for every positive integer n.**

**Q20. If** $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$**, then prove that**

**(i)** $A_\alpha . A_\beta = A_{\alpha+\beta}$ **(ii)** $(A_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$**, for every positive integer n.**

**Q21. If 'a' is a non-zero real or complex number. Use the principle of mathematical induction to prove that**

**If** $A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}$**, then** $A^n = \begin{bmatrix} a^n & na^{n-1} \\ 0 & a^n \end{bmatrix}$ **for every positive integer n.**

**Q22. Under what condition is the matrix equation** $A^2 - B^2 = (A-B)(A+B)$ **is true?**

**Q23. If** $AB = A$ **and** $BA = B$**, then show that** $A^2 = A, B^2 = B$**.**

**Q24.If** $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ **is a matrix satisfying** $AA^T = 9I_3$**, then find the values of 'a' and 'b'.**

**(ANS:** $a = -2$ **and** $b = -1$**)**

**Q25. Find the values of x, y, z if the matrix** $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ **satisfy the equation** $A^T A = I_3$**.**

**(ANS:** $x = \pm \frac{1}{\sqrt{2}}, y = \pm \frac{1}{\sqrt{6}}, z = \pm \frac{1}{\sqrt{3}}$**)**

# ASSIGNMENT
# CH-DETERMINANTS

**Q1. For what value of 'x' the matrix** $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix}$ **is singular?**

**(ANS:** $x = -1$**)**

**Q2. Determine the values of x for which the matrix**

$A = \begin{bmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{bmatrix}$ **is singular. (ANS:** $x = 0, \frac{1}{2}\left(3 \pm \sqrt{205}\right)$ **)**

**Q3. If** $[\cdot]$ **notes the greatest integer less than or equal to the real number under consideration, and** $-1 \le x < 0,\ 0 \le y < 1,\ 1 \le z < 2$**, then find the value of the determinant**

$$\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$$

**(ANS: 1)**

**Q4. If** $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$**, find the determinant of the matrix** $A^2 - 2A$**. (ANS: 25)**

**Q5. Find the minors and cofactors of elements of the matrix**

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{bmatrix}$$

**(ANS:** $\begin{matrix} M_{11} = -40, M_{12} = -10, M_{13} = 35 \\ M_{21} = 16, M_{22} = 8, M_{23} = -4 \\ M_{31} = 8, M_{32} = 14, M_{33} = -17 \end{matrix}$ **)**

**Q6. Prove that the determinant** $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ **is independent of** $\theta$ **.**

**Q7. Let** $\begin{vmatrix} 3 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$**. Find possible values of x and y if x, y are natural numbers.**

**(ANS:** $(1,2);(2,4);(4,2);(8,1)$ **)**

**Q8. Evaluate** $\begin{vmatrix} \cos 15^o & \sin 15^o \\ \sin 75^o & \cos 75^o \end{vmatrix}$.

**Q9. Without expanding evaluate the following determinants.**

**(i)** $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$ **(ii)** $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ **(iii)** $\begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix}$

**(iv)** $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$ **(v)** $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$ **(vi)** $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$

**(vii)** $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix}$ **(viii)** $\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (a^y+a^{-y})^2 & (a^y-a^{-y})^2 & 1 \\ (a^z+a^{-z})^2 & (a^z-a^{-z})^2 & 1 \end{vmatrix}$

**(ix)** $\begin{vmatrix} 265 & 240 & 219 \\ 240 & 225 & 198 \\ 219 & 198 & 181 \end{vmatrix}$ **(x)** $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix} = 0$

**Q10. Prove the following:**

**(i)** $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

**(ii)** $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

**(iii)** $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$

**(iv)** $\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$

**(v)** $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

**(vi)** $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2.b^2.c^2$

**(vii)** $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

**(viii)** $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$

**(ix)** $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

**(x)** $\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2-ac)(ax^2+2bxy+cy^2)$

**(xi)** $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$

**(xii)** $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$

**(xiii)** $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

**(xiv)** $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

**(xv)** $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$

**(xvi)** $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$

**(xvii)** $\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)(a^2+b^2+c^2)$

**(xviii)** $\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$

**(xix)** $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$

**(xx)** $\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = abc(a^2+b^2+c^2)^3$

**(xxi)** $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$

**(xxii)** $\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix} = (a^3+b^3)^2$

**(xxiii)** $\begin{vmatrix} a & b-c & c-b \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix} = (a+b-c)(b+c-a)(c+a-b)$

**(xxiv)** $\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix} = 4abc$

**Q11. If a,b,c are in A.P. then show that**

**(i)** $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$ **(ii)** $\begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix} = 0$

**Q12. Solve the following:**

**(i)** $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$ **(ii)** $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

**(iii)** $\begin{vmatrix} 15-2x & 11-3x & 7-x \\ 11 & 17 & 14 \\ 10 & 16 & 13 \end{vmatrix} = 0$ **(iv)** $\begin{vmatrix} 1 & x & x^2 \\ 1 & a & a^2 \\ 1 & b & b^2 \end{vmatrix} = 0, a \neq b$

**(v)** $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$

**Q13. If a, b, c are all positive and are p^th^, q^th^,r^th^ terms of a G.P., then show that:**

$$\Delta = \begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$$

**Q14. If a, b, c are positive and unequal, show that the value of the determinant** $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ **is always negative.**

**Q15. Show that the points** $(a, b+c), (b, c+a)$ and $(c, a+b)$ **are collinear.**

**Q16. If the points** $(a,0), (0,b)$ and $(1,1)$ **are collinear, prove that** $a+b=ab$**.**

**Q18. If** $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$**, find** $adjA$ **and verify that** $A(adjA) = (adjA)A = |A| I_3$

**Q19. Find the inverse of** $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ **and verify that** $A^{-1}A = I_3$**.**

**Q20. If** $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$**, show that** $A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$**.**

**Q21. If** $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ **and** $B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$**, verify that** $(AB)^{-1} = B^{-1}A^{-1}$**.**

**Q22. Show that** $A=\begin{bmatrix}2 & -3\\3 & 4\end{bmatrix}$ **satisfies the equation** $x^2-6x+17=0$**. Hence find** $A^{-1}$**.** $\left(ANS: A^{-1}=\frac{1}{17}\begin{bmatrix}4 & 3\\-3 & 2\end{bmatrix}\right)$

**Q23. For the matrix** $A=\begin{bmatrix}3 & 1\\7 & 5\end{bmatrix}$**, find x and y so that** $A^2+xI=yA$**. Hence, find** $A^{-1}$**.** $\left(ANS: x=8 \text{ and } y=8, A^{-1}=\begin{bmatrix}5/8 & -1/8\\-7/8 & 3/8\end{bmatrix}\right)$

**Q24. Show that the matrix** $A=\begin{bmatrix}1 & 2 & 2\\2 & 1 & 2\\2 & 2 & 1\end{bmatrix}$ **satisfies the equation** $A^2-4A-5I_3=O$ **and hence find** $A^{-1}$. **(ANS:** $A^{-1}=\frac{1}{5}\begin{bmatrix}-3 & 2 & 2\\2 & -3 & 2\\2 & 2 & -3\end{bmatrix}$ **)**

**Q25. Find the matrix A satisfying the matrix equation**

$$\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}A\begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$\left(ANS: A=\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}\right)$

**Q26. Find the inverse of the matrix** $A=\begin{bmatrix}a & b\\c & \frac{1+bc}{a}\end{bmatrix}$ **and show that** $aA^{-1}=\left(a^2+bc+1\right)I-aA.$

**Q27. If** $A=\begin{bmatrix}4 & 5\\2 & 1\end{bmatrix}$**, then show that** $A-3I=2\left(I+3A^{-1}\right)$

**Q28. Given** $A=\begin{bmatrix}5 & 0 & 4\\2 & 3 & 2\\1 & 2 & 1\end{bmatrix}$**,** $B^{-1}=\begin{bmatrix}1 & 3 & 3\\1 & 4 & 3\\1 & 3 & 4\end{bmatrix}$**. Compute** $(AB)^{-1}$**.**

$\left(ANS: \begin{bmatrix}-2 & 19 & -27\\-2 & 18 & -25\\-3 & 29 & -42\end{bmatrix}\right)$**.**

**Q29. Let** $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **and** $G(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$ **then show that**

**(i)** $\left[F(\alpha)\right]^{-1} = F(-\alpha)$ **(ii)** $\left[G(\beta)\right]^{-1} = G(-\beta)$

**(iii)** $\left[F(\alpha)G(\beta)\right]^{-1} = G(-\beta)F(-\alpha)$

**Q30. Show that:**

$$\begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

**Q31. The sum of three numbers is 6. If we multiply the third number by 2 and add the first number to the result, we get 7. By adding second and third numbers to three times the first number we get 12. Use matrix method to find the numbers. (ANS: 3, 1, 2)**

**Q32. If** $f(x) = ax^2 + bx + c$ **is a quadratic function such that** $f(1) = 8$**,** $f(2) = 11$ **and** $f(-3) = 6$**, find** $f(x)$ **by using matrices. Also find** $f(0)$ **.**

**Q33. Solve the following systems of equations by using matrices:**

**(i)** $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4,\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$ and $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$ **(ANS: 2, 3, 5)**

**(ii)** $3x + y + z = 2, 2x - 4y + 3z = -1, 4x + y - 3z = -11$ **(ANS: -1, 2, 3)**

**(iii)** $x - 4y - z = 11,\ 2x - 5y + 2z = 39, -3x + 2y + z = 1$ **(ANS:-1, -5, 8)**

**(iv)** $6x + y - 3z = 5, x + 3y - 2z = 5, 2x + y = 4z = 8$ **(ANS: 1,2,1)**

**(v)** $x + y = 5, y + z = 3, x + z = 4\ (ANS : x = 3, y = 2, z = 1)$

**(vi)** $2y - 3z = 0, x + 3y = -4, 3x + 4y = 3 (ANS : x = 5, y = -3, z = -2)$

**(vii)** $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2\ (ANS : x = 2, y = 3, z = 5)$

**(viii)** $5x - 7y + z = 11,\ 6x - 8y - z = 15,\ 3x + 2y - 6z = 7\ (ANS : x = 1, y = -1, z = -1)$

**Q34. If** $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$**, find** $A^{-1}$ **and hence solve the system of linear equations** $x + 2y + z = 4,\ -x + y + z = 0, x - 3y + z = 2$**.**

$(ANS : x = 9/5, y = 2/5, z = 7/5)$

**Q35. Determine the product** $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ **and use it to solve the system of equations:**

$$x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1$$

**(ANS:** $x = 3, y = -2$ and $z = -1$**)**

**Q36. An amount of Rs 5000 is put into three investments at the rate of interest of 6%, 7% and 8% per annum respectively. The total annual income is Rs358. If the combined income from the first two investments is Rs70more than the income from the third, find the amount of each investment by matrix method. (ANS: 1000,2200,1800).**

# ASSIGNMENT
## CH – CONTINUITY & DIFFERENTIABILTY

**CONTINUITY**

**Q1. Test the continuity of the following function at the indicated points**

**(i)** $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ **at x = 0.**

**(ii)** $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ 2, & x = 0 \end{cases}$ **at x = 0.**

**(iii)** $f(t) = \begin{cases} \frac{\cos t}{\pi/2 - t}, & t \neq \pi/2 \\ 1, & t = \pi/2 \end{cases}$ **at** $t = \pi/2$.

**(iv)** $f(x) = \begin{cases} 1/2 - x; & 0 \leq x < 1/2 \\ 1; & x = 1/2 \\ 3/2 - x; & 1/2 < x \leq 1 \end{cases}$ **at** $x = 1/2$

**(v)** $f(x) = \begin{cases} 2 - x, & x < 2 \\ 2 + x, & x > 2 \end{cases}$ **at x = 2**

**(vi)** $f(x) = |x - 1| + |x - 2|$ **at x =1 and x = 2**

**Q2. Determine the value of 'k' for which the following function is continuous at the indicated points.**

**(i)** $f(x) \begin{cases} \frac{x^2 - 9}{x - 3}, & x \neq 3 \\ k, & x = 3 \end{cases}$ **at x = 3. (ANS:** $k = 6$**)**

**(ii)** $f(x) = \begin{cases} \frac{x^2 - 2x - 3}{x + 1}, & x \neq -1 \\ \text{k}, & x = -1 \end{cases}$ **at x = -1. (ANS:** $k = -4$**)**

**(iii)** $f(x) = \begin{cases} \frac{1 - \cos 2x}{2x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$ **at x = 0. (ANS:** $k = 1$**)**

**(iv)** $f(x) = \begin{cases} 2x - 1, & x < 2 \\ \text{k}, & x = 2 \\ x + 1, & x > 2 \end{cases}$ **at x = 2. (ANS:** $k = 3$**)**

**(v)** $f(x) = \begin{cases} \frac{\sin^2 kx}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ **at x = 0. (ANS:** $k = \pm 1$

P.T.O.

**(vi)** $f(x) = \begin{cases} 3ax + b, & if\ x > 1 \\ 11 \quad , & if\ x = 1 \\ 5ax - 2b\ , & if\ x < 1 \end{cases}$ **at x = 1, find 'a' and 'b'. (ANS:** $a = 3; b = 2$**)**

**(vii)** $f(x) = \begin{cases} \dfrac{1 - \cos 4x}{x^2}, & \text{if } x < 0 \\ k & , \text{if } x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}, & \text{if } x > 0 \end{cases}$ **at x = 0. (ANS:** $k = 8$**)**

**(ix)** $f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & x \neq \pi/2 \\ 3 & , x = \pi/2 \end{cases}$ **at** $x = \pi/2$ **(ANS:** $k = 6$**)**

**(x)** $f(x) = \begin{cases} \dfrac{1 - \cos kx}{x \sin x}, & x \neq 0 \\ \dfrac{1}{2} & , x = 0 \end{cases}$ **at x = 0 (ANS:** $k = \pm 1$**)**

**(xi)** $f(x) = \begin{cases} \dfrac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2 + 1} - 1}, & x \neq 0 \\ k & , x = 0 \end{cases}$ **at x = 0. (ANS:** $k = -4$**)**

**(xii)** $f(x) = \begin{cases} \dfrac{1 - \cos 4x}{8x^2}, & x \neq 0 \\ \text{k} & , x = 0 \end{cases}$ **at x = 0. (ANS:** $k = 1$**)**

**(xiii)** $f(x) = \begin{cases} (x - 1)\tan \dfrac{\pi x}{2}, & x \neq 1 \\ k & , x = 1 \end{cases}$ **at x = 1. (ANS:** $k = \dfrac{-2}{\pi}$**)**

**(xiv)** $f(x) = \begin{cases} 2x^2 + k, & if\ x \geq 0 \\ -2x^2 + k, & if\ x < 0 \end{cases}$ **at x = 0. (ANS:** $k \text{ is any real number}$**)**

**(xv)** $f(x) = \begin{cases} \dfrac{\sin(a + 1)x + \sin x}{x}, & x < 0 \\ c & , x = 0 \\ \dfrac{\sqrt{x + bx^2} - \sqrt{x}}{bx\sqrt{x}} & , x > 0 \end{cases}$ **. (ANS:** $a = -\dfrac{3}{2},\ b \in R - \{0\}\ , c = \dfrac{1}{2}$**)**

**(xvi)** $f(x)=\begin{cases}\dfrac{\sqrt{1+px}-\sqrt{1-px}}{x}, & -1\le x<0\\ \dfrac{2x+1}{x-2}, & 0\le x\le 1\end{cases}$ **is continuous in the interval** $[-1,1]$ **then find 'p'. (ANS:** $p=-1/2$**)**

**Q3. Prove that the greatest integer function** $[x]$ **is continuous at all points except at integer points.**

**Q4.Let** $f(x+y)=f(x)+f(y)$ **for all** $x,y\in R$**.If** $f(x)$ **is continuous at** $x=0$**, show that** $f(x)$ **is continuous at all** $x$**.**

**Q5. Discuss the continuity of the following functions:**

**(i)** $f(x)=\begin{cases}\dfrac{|x-4|}{x-4}, & x\neq 4\\ 0, & x=4\end{cases}$

**(ii)** $f(x)=\begin{cases}\dfrac{\sin 2x}{x}, & x<0\\ x+2, & x\ge 0\end{cases}$

**Q6. Find the values of 'a' and 'b' so that the function** $f(x)$ **defined by**

$$f(x)=\begin{cases}x+a\sqrt{2}\sin x, & if\ \ 0\le x\pi/4\\ 2x\cot x+b, & if\ \ \pi/4x<\pi/2\\ a\cos 2x-b\sin x, & if\ \ \pi/2\le x\le\pi\end{cases}$$

**becomes continuous on** $[0,\pi]$**. (ANS:** $a=\pi/6,\ b=-\pi/12$**)**

**Q7. The function** $f(x)$ **is defined as follows:**

$$f(x)=\begin{cases}x^2+ax+b, & 0\le x<2\\ 3x+2, & 2\le x\le 4\\ 2ax+5b, & 4<x\le 8\end{cases}$$

**if** $f$ **is continuous on** $[0,8]$**, find the values of 'a' and 'b'.**
**(ANS:** $a=3, b=-2$**)**

**Q8. If** $f(x)=\dfrac{\tan\left(\dfrac{\pi}{4}-x\right)}{\cot 2x}$ **for** $x\neq\pi/4$**,find the value which can be assigned to** $f(x)$ **at** $x=\pi/4$ **so that the function becomes continuous every where in** $[0,\pi/2]$**. (ANS:** $f(\pi/4)=1/2$**)**

**Q9. Show that the function** $g(x)=x-[x]$ **is discontinuous at all integral points. Here** $[x]$ **denotes the greatest integer function.**

**Q10. Show that** $f(x) = \cos x^2$ **is a continuous function.**

## DIFFERNTIATION

**Q11.Show that** $f(x) = |x|$ **is not differentiable at** $x = 0$**.**

**Q12.Show that the function** $f(x) = \begin{cases} x - 1, & if x < 2 \\ 2x - 3, & if x \geq 2 \end{cases}$ **is not differentiable at x = 2.**

**Q13.Show that** $f(x) = x^{1/3}$ **is not differentiable at x = 0.**

**Q14. Show that** $f(x) = |x - 2|$ **is continuous but not differentiable at x = 2.**

**Q15. Give an example of a function which is everywhere continuous but fails to be differentiable exactly at two points.**

**Q16. Show that** $f(x) = x^2$ **is differentiable at x = 1 and find** $f'(1)$**.**

**Q17. If** $f(x)$ **is differentiable at x = a, find** $\lim_{x \to a} \frac{x^2 f(a) - a^2 f(x)}{x - a}$ **.**

**(ANS:** $2af(a) - a^2 f'(a)$**)**

**Q18. If** $f(2) = 4$ **and** $f'(2) = 1$**, then find** $\lim_{x \to 2} \frac{xf(2) - 2f(x)}{x - 2}$**. (ANS:** $2$**)**

**Q19. For what choice of 'a' and 'b' is the function** $f(x) = \begin{cases} x^2 & , x \leq c \\ ax + b & , x > c \end{cases}$ **is differentiable at x = c. (ANS:** $a = 2c, b = -c^2$**)**

**Q20. Discuss the differentiability of** $f(x) = x|x|$ **at x = 0.**

**Q21. Show that the function** $f(x) = \begin{cases} x \sin \frac{1}{x}, & when\ x \neq 0 \\ 0 & , when\ x = 0 \end{cases}$ **is continuous but not differentiable at x = 0.**

**Q22. If** $f(x) = \begin{cases} ax^2 - b, & \text{if } |x| < 1 \\ \frac{1}{|x|} & , if\ |x| \geq 1 \end{cases}$ **is differentiable at x = 1, find a, b.**

**(ANS:** $a = -\frac{1}{2}, b = -\frac{3}{2}$**)**

**Q23. If** $f(x) = \begin{cases} x^2 + 3x + a, & for\ x \leq 1 \\ bx + 2, & for\ x > 1 \end{cases}$ **is everywhere differentiable, find the values of 'a' and 'b'. (ANS:** $a = 3, b = 5$**)**

**Q24. Differentiate the following w.r.t 'x':**

**(i)** $\sin\left(x^2+1\right)$ **(ii)** $e^{\sin x}$ **(iii)** $\log\left(\sin x\right)$ **(iv)** $\sqrt{x^2+x+1}$

**(v)** $\sin^3 x$ **(vi)** $\dfrac{1}{\sqrt{a^2-x^2}}$ **(vii)** $\log\left(\sec x+\tan x\right)$

**(viii)** $e^{x\sin x}$ **(ix)** $\sin^{-1}\left(x^3\right)$ **(x)** $\sin^{-1}\left(\dfrac{a+b\cos x}{b+a\cos x}\right), b>a$ **(xi)** $e^{e^x}$

**(xii)** $\log_7\left(\log_7 x\right)$ **(xiii)** $\log_x 2$ **(xiv)** $\sec\left(\log x^n\right)$ **(xv)** $\log\tan\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)$

**(xvi)** $\sqrt{\log\left\{\sin\left(\dfrac{x^2}{3}-1\right)\right\}}$ **(xvii)** $\log\left(x+\sqrt{a^2+x^2}\right)$ **(xviii)** $\log\left(\dfrac{a+b\sin x}{a-b\sin x}\right)$

**(xix)** $\dfrac{e^x+e^{-x}}{e^x-e^{-x}}$ **(xx)** $\log\sqrt{\dfrac{1+\sin x}{1-\sin x}}$ **(xxi)** $\sin\left(m\sin^{-1}x\right)$ **(xxii)** $a^{\left(\sin^{-1}x\right)^2}$

**(xxiii)** $e^{\cos^{-1}\left(\sqrt{1-x^2}\right)}$ **(xxiv)** $\log\left(\dfrac{x^2+x+1}{x^2-x+1}\right)$ **(xxv)** $\dfrac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}}$

| **ANSWERS** |
|---|
| **(i)** $2x\cos\left(x^2+1\right)$ **(ii)** $\cos x\, e^{\sin x}$ **(iii)** $\cot x$ **(iv)** $\dfrac{2x+1}{2\sqrt{x^2+x+1}}$ |
| **(v)** $3\sin^2 x.\cos x$ **(vi)** $\dfrac{x}{\left(a^2-x^2\right)^{3/2}}$ **(vii)** $\sec x$ **(viii)** $e^{x\sin x}\left(x\cos x+\sin x\right)$ |
| **(ix)** $\dfrac{3x^2}{\sqrt{1-x^6}}$ **(x)** $\dfrac{-\sqrt{b^2-a^2}}{b+a\cos x}$ **(xi)** $e^x.e^{e^x}$ **(xii)** $\dfrac{1}{x\log_7 x\left(\log_e 7\right)^2}$ |
| **(xiii)** $-\dfrac{1}{\left(\log_2 x\right)^2}\times\dfrac{1}{x\log_e 2}$ **(xiv)** $\dfrac{n}{x}\times\sec\left(\log x^n\right)\tan\left(\log x^n\right)$ **(xv)** $\sec x$ |
| **(xvi)** $\dfrac{x\cot\left(\dfrac{x^2}{3}-1\right)}{3\sqrt{\log\left\{\sin\left(\dfrac{x^2}{3}-1\right)\right\}}}$ **(xvii)** $\dfrac{1}{\sqrt{a^2+x^2}}$ **(xviii)** $\dfrac{2ab\cos x}{a^2-b^2\sin^2 x}$ |
| **(xix)** $-\dfrac{4}{\left(e^x-e^{-x}\right)^2}$ **(xx)** $\sec x$ **(xxi)** $\dfrac{m}{\sqrt{1-x^2}}\cos\left(m\sin^{-1}x\right)$ |
| **(xxii)** $\dfrac{2\log a.\sin^{-1}x}{\sqrt{1-x^2}}\times a^{\left(\sin^{-1}x\right)^2}$ **(xxiii)** $e^{\cos^{-1}\sqrt{1-x^2}}\times\dfrac{1}{\sqrt{1-x^2}}$ **(xxv)** $-\dfrac{2\left(x^2-1\right)}{x^4+x^2+1}$ |
| **(xxvi)** $2x+\dfrac{2x^3}{\sqrt{x^4-1}}$ |

**Q25. Find** $\frac{dy}{dx}$ **if (i)** $y=\frac{e^x+\log x}{\sin 3x}$ **(ii)** $y=\frac{\sin x+x^2}{\cot 2x}$

**(ANS:** $e^x\left\{\frac{2x}{1+x^2}+\log\left(1+x^{2s}\right)\right\}$ **;** $2\left(\sin x+x^2\right)\sec^2 2x+\left(\cos x+2x\right)\tan 2x$**)**

**Q26. If** $y=\left[x+\sqrt{x^2+a^2}\right]^n$**, then prove that** $\frac{dy}{dx}=\frac{ny}{\sqrt{x^2+a^2}}$**.**

**Q27. If** $y=\frac{x\sin^{-1}x}{\sqrt{1-x^2}}+\log\sqrt{1-x^2}$**, then prove that** $\frac{dy}{dx}=\frac{\sin^{-1}x}{\left(1-x^2\right)^{3/2}}$**.**

**Q28. If** $y=\frac{\sqrt{a^2+x^2}+\sqrt{a^2-x^2}}{\sqrt{a^2+x^2}-\sqrt{a^2-x^2}}$**, show that** $\frac{dy}{dx}=\frac{-2a^2}{x^3}\left\{1+\frac{a^2}{\sqrt{a^4-x^4}}\right\}$**.**

**Q29. If** $y=\sqrt{\frac{1-x}{1+x}}$**, prove that** $\left(1-x^2\right)\frac{dy}{dx}+y=0$**.**

**Q30. If** $y=\sqrt{\frac{1+e^x}{1-e^x}}$**, show that** $\frac{dy}{dx}=\frac{e^x}{\left(1-e^x\right)\sqrt{1-e^{2x}}}$

**Q31. If** $y=\log\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$**, prove that** $\frac{dy}{dx}=\frac{-1}{2\sqrt{x^2-1}}$**.**

**Q32. If** $y=\sqrt{x+1}+\sqrt{x-1}$**, prove that** $\sqrt{x^2-1}\frac{dy}{dx}=\frac{1}{2}y$**.**

**Q33. If** $y=\log\left\{\sqrt{x-1}-\sqrt{x+1}\right\}$**, show that** $\frac{dy}{dx}=\frac{-1}{2\sqrt{x^2-1}}$**.**

**Q34. If** $y=\frac{x}{x+2}$**, prove that** $x\frac{dy}{dx}=(1-y)\,y$**.**

**Q35. If** $y=\frac{x\sin^{-1}x}{\sqrt{1-x^2}}$**, prove that** $\left(1-x^2\right)\frac{dy}{dx}=x+\frac{y}{x}$**.**

**Q36. If** $xy=4$**, prove that** $x\left(\frac{dy}{dx}+y^2\right)=3y$**.**

**Q37. Prove that** $\frac{d}{dx}\left\{\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\}=\sqrt{a^2-x^2}$**.**

**Differentiation of inverse trigonometric functions:**
**Differentiate the following**

| | |
|---|---|
| **Q38.** $\tan^{-1}\left\{\frac{1-\cos x}{\sin x}\right\}$ **(ANS:** $1/2$**).** | **Q43.** $\tan^{-1}\left\{\sqrt{1+x^2}+x\right\}$ **(ANS:** $\frac{1}{2\left(1+x^2\right)}$**)** |
| **Q39.** $\tan^{-1}\left\{\sqrt{\frac{1-\cos x}{1+\cos x}}\right\}$ **(ANS:** $1/2$**)** | **Q44.** $\tan^{-1}\left\{\sqrt{1+x^2}-x\right\}$ **(ANS:** $-\frac{1}{2\left(1+x^2\right)}$**)** |

| | |
|---|---|
| **Q40.** $\tan^{-1}\left\{\frac{\cos x}{1+\sin x}\right\}$ **(ANS:** $-1/2$**)** | **Q45.** $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}, x \neq 0$ |
| **Q41.** $\tan^{-1}\left\{\sqrt{\frac{1+\sin x}{1-\sin x}}\right\}$ **(ANS:** $1/2$**)** | **(ANS: Q45:** $\frac{1}{2}\left(\frac{1}{1+x^2}\right)$**)** |
| **Q42.** $\tan^{-1}(\sec x+\tan x)$ **(ANS:** $1/2$**)** | |

**Q46.** $\tan^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}$ **(ANS:** $1/2$**)**

**Q47.** $\tan^{-1}\left(\frac{a+x}{1-ax}\right)$ **(ANS:** $\frac{1}{1+x^2}$**)**

**Q48.** $\tan^{-1}\left(\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right)$ **(Ans: -1)**

**Q49.** $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$ **(ANS:** $\frac{3a}{a^2+x^2}$**)**

**Q50.** $\tan^{-1}\sqrt{\frac{a-x}{a+x}}$ **(ANS:** $-\frac{1}{2\sqrt{a^2-x^2}}$**)**

**Q51.** $\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ **(ANS:** $\frac{4}{1+x^2}$**)**

**Q52.** $\sin^{-1}x+\sin^{-1}\sqrt{1-x^2}$ **(ANS: 0)**

**Q53.** $\sin^{-1}\left\{\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right\}$ **(ANS:** $-\frac{1}{2\sqrt{1-x^2}}$**)**

**Q54.** $\tan^{-1}\left(\frac{2^{x+1}}{1-4^x}\right)$ **(ANS:** $\frac{2^{x+1}\log_e 2}{1+4^x}$**)**

**Q55.** $\tan^{-1}\left(\frac{\sqrt{x}+\sqrt{a}}{1-\sqrt{xa}}\right)$ **(ANS:** $\frac{1}{2\sqrt{x}(1+x)}$**)**

**Q56.** $\tan^{-1}\left(\frac{a+bx}{b-ax}\right)$ **(ANS:** $\frac{1}{1+x^2}$**)**

**Q57.** $\tan^{-1}\left(\frac{x}{1+6x^2}\right)$ **(ANS:** $\frac{3}{1+9x^2}-\frac{2}{1+4x^2}$**)**

**Q58.** $\tan^{-1}\left(\frac{4x}{1-4x^2}\right)$ **(ANS:** $\frac{4}{1+4x^2}$**)**

**Q59.** $\tan^{-1}\left\{\frac{x^{1/3}+a^{1/3}}{1-(ax)^{1/3}}\right\}$ **(ANS:** $\frac{1}{3}.\frac{x^{-2/3}}{1+x^{2/3}}$**)**

**Q60.** $\sin\left[2\tan^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right\}\right]$ **(ANS:** $\frac{-x}{\sqrt{1-x^2}}$**)**

**Q61.** $\cot^{-1}\left(\frac{1-x}{1+x}\right)$ **(ANS:** $\frac{1}{1+x^2}$**)**

**Differentiation of implicit functions**

**Q62. If** $x^2+2xy+y^3=42$**, find** $\frac{dy}{dx}$ **(ANS:** $\frac{dy}{dx}=-\frac{2(x+y)}{2x+y^2}$

**Q63. If** $x^3+y^3=3axy$**, find** $\frac{dy}{dx}$ **(ANS:** $\frac{dy}{dx}=\frac{ay-x^2}{y^2-ax}$**))**

**Q64. If** $\log\left(x^2+y^2\right)=2\tan^{-1}\left(\frac{y}{x}\right)$**, show that** $\frac{dy}{dx}=\frac{x+y}{x-y}$

**Q65.** ***If*** $x\sqrt{1+y}+y\sqrt{1+x}=0$***, prove that*** $\frac{dy}{dx}=-\frac{1}{(x+1)^2}$

**Q66.** ***If*** $\cos\left(\frac{x^2-y^2}{x^2+y^2}\right)=\tan^{-1}a$***, prove that*** $\frac{dy}{dx}=\frac{y}{x}$**.**

**Q67.** ***If*** $\sqrt{1-x^6}+\sqrt{1-y^6}=a\left(x^3-y^3\right)$***, prove that*** $\frac{dy}{dx}=\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$**.**

**Q68.** ***If*** $x^2+y^2=t-\frac{1}{t}$ ***and*** $x^4+y^4=t^2+\frac{1}{t^2}$ ***then prove that*** $\frac{dy}{dx}=\frac{1}{x^3y}$**.**

**Q69.** ***If*** $y=b\tan^{-1}\left(\frac{x}{a}+\tan^{-1}\frac{y}{x}\right)$***,find*** $\frac{dy}{dx}$ ***(ANS:*** $\frac{\frac{1}{a}-\frac{y}{x^2+y^2}}{\frac{1}{b}\frac{\sec^2 y}{b}-\frac{x}{x^2+y^2}}$***)***

**Q70. If** $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ **, prove that** $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

**Q71. If** $y\sqrt{1-x^2}+x\sqrt{1-y^2}=1$**, prove that** $\frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}$

**Q72. If** $xy=1$**, prove that** $\frac{dy}{dx}+y^2=0$

**Q73. If** $x\sqrt{1+y}+y\sqrt{1+x}=0$**, prove that** $\left(1+x^2\right)\frac{dy}{dx}+1=0$**.**

**Q74. If** $xy\log(x+y)=1$**, prove that** $\frac{dy}{dx}=-\frac{y\left(x^2y+x+y\right)}{x\left(xy^2+x+y\right)}$**.**

**Q75. If** $y = x\sin(a+y)$ **, prove that** $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y) - y\cos(a+y)}$ **.**

**Q76. If** $x\sin(a+y) + \sin a\cos(a+y) = 0$ **, prove that** $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ **.**

**Q77. If** $y = x\sin y$ **, prove that** $\frac{dy}{dx} = \frac{y}{x(1 - x\cos y)}$ **.**

**Q78. If** $\cos y = x\cos(a+y)$ **, with** $\cos a \neq \pm 1$ **, prove that** $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$ **.**

**Q79. If** $e^x + e^y = e^{x+y}$ **, prove that** $\frac{dy}{dx} = \frac{e^x(e^y - 1)}{e^y(e^x - )}$ **.**

**Q80. If** $\tan(x+y) + \tan(x-y) = 1$ **, find** $\frac{dy}{dx}$ **(ANS:** $\frac{\sec^2(x-y) + \sec^2(x+y)}{\sec^2(x-y) - \sec^2(x+y)}$ **)**

## LOGARITHMIC DIFFERENTIATION

**Q81. Differentiate the following:**

**(i)** $x^{\sqrt{x}}$ **(ii)** $x^{\cos^{-1}x}$ **(iii)** $(\sin x)^{\cos^{-1}x}$ **(iv)** $x^{x^x}$ **(v)** $(x^x)^x$

**(vi)** $(\tan x)^{1/x}$ **(vii)** $\sin(x^x)$ **(viii)** $(\sin^{-1}x)^x$

**ANSWERS**

**(i)** $x^{\sqrt{x}}\left(\frac{\log x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}\right)$ **(ii)** $x^{\cos^{-1}x}\left\{\frac{-\log x}{\sqrt{1-x^2}} + \frac{\cos^{-1}x}{x}\right\}$

**(iii)** $(\sin x)^{\cos^{-1}x}\left\{\cos^{-1}x.\cot x - \frac{\log\sin x}{\sqrt{1-x^2}}\right\}$ **(iv)** $x^{x^x}.x^x\left\{(1+\log x).\log x + \frac{1}{x}\right\}$

**(v)** $x.x^{x^2}(2\log x + 1)$ **(vi)** $(\tan x)^{1/x}\left\{-\frac{1}{x^2}\log\tan x + \frac{1}{x}\frac{\sec^2 x}{\tan x}\right\}$

**(vii)** $x^x(1+\log x)\cos(x^x)$ **(viii)** $(\sin^{-1}x)^x\left\{\log\sin^{-1}x + \frac{x}{\sin^{-1}x}.\frac{1}{\sqrt{1-x^2}}\right\}$

**Q82. Differentiate the following:**

**(i)** $x^{\cot x} + \frac{2x^2 - 3}{x^2 + x + 2}$ **(ANS:** $x^{\cot x}\left\{-\cos ec^2 x.\log x + \frac{\cot x}{x}\right\} + \frac{2x^2 + 14x + 3}{(x^2 + x + 2)^2}$ **)**

**(ii)** $\log(x^x + \cos ec^2 x)$ **(ANS:** $\frac{1}{x^x + \cos ec^2 x}\left\{x^x(1+\log x) - 2\cos ec^2 x\cot x\right\}$ **)**

**(iii)** $x^x e^{2(x+3)}$ **(ANS:** $x^x e^{2(x+3)}(3 + \log x)$ **)**

**(iv)** $x^{x\cos x} + \frac{x^2 + 1}{x^2 - 1}$ **(ANS:** $x^{x\cos x}\left\{(1+\log x)\cos x + (1 - x\log x)\sin x\right\} - \frac{4x}{(x^2+1)^2}$ **)**

**(v)** $\left(x\cos x\right)^{x}+\left(x\sin x\right)^{1/x}$

**(ANS:** $\left(x\cos x\right)^{x}\left\{1-x\tan x+\log\left(x\cos x\right)\right\}+\left(x\sin x\right)^{1/x}\dfrac{\left\{1+x\cot x-\log\left(x\sin x\right)\right\}}{x^{2}}$ **)**

**Q83. If** $y=a^{x}+e^{x}+x^{x}+x^{a}$ **find** $\dfrac{dy}{dx}$ **at x = a**

**(ANS:** $\left(\dfrac{dy}{dx}\right)_{x=a}=e^{a}+2a^{a}\left(1+\log a\right)$ **)**

**Q84. If** $y=\dfrac{\sqrt{1-x^{2}}\left(2x-3\right)^{1/2}}{\left(x^{2}+2\right)^{2/3}}$ **, find** $\dfrac{dy}{dx}$

**(ANS:** $\dfrac{dy}{dx}=\dfrac{\sqrt{1-x^{2}}\left(2x-3\right)^{1/2}}{\left(x^{2}+2\right)^{2/3}}\left\{-\dfrac{x}{1-x^{2}}+\dfrac{1}{2x-3}-\dfrac{4x}{3\left(x^{2}+2\right)}\right\}$ **)**

**Q85. Find the derivative of** $\dfrac{\sqrt{x}\left(x+4\right)^{3/2}}{\left(4x-3\right)^{4/3}}$ **w.r.t. x.**

**(ANS:** $\dfrac{\sqrt{x}\left(x+4\right)^{3/2}}{\left(4x-3\right)^{4/3}}\left\{\dfrac{1}{2x}+\dfrac{3}{2\left(x+4\right)}-\dfrac{16}{3\left(4x-3\right)}\right\}$ **)**

**Q86. If** $x^{m}y^{n}=\left(x+y\right)^{m+n}$**, prove that** $\dfrac{dy}{dx}=\dfrac{y}{x}$**.**

**Q87. If** $y=\dfrac{ax^{2}}{\left(x-a\right)\left(x-b\right)\left(x-c\right)}+\dfrac{bx}{\left(x-b\right)\left(x-c\right)}+\dfrac{c}{x-c}+1$**, prove that** $\dfrac{dy}{dx}=\dfrac{y}{x}\left\{\dfrac{a}{a-x}+\dfrac{b}{b-x}+\dfrac{c}{c-x}\right\}$**.**

**Q88. Given that** $\cos\dfrac{x}{2}.\cos\dfrac{x}{4}.\cos\dfrac{x}{8}...=\dfrac{\sin x}{x}$**, prove that** $\dfrac{1}{2^{2}}\sec^{2}\dfrac{x}{2}+\dfrac{1}{2^{4}}\sec^{2}\dfrac{x}{4}+...=\operatorname{cosec}^{2}x-\dfrac{1}{x^{2}}$**.**

**Q89. If** $x^{16}y^{9}=\left(x^{2}+y\right)^{17}$**, prove that** $x\dfrac{dy}{dx}=2y$**.**

**Q90. If** $y=\dfrac{e^{ax}.\sec x.\log x}{\sqrt{1-2x}}$ **then prove that**

$\dfrac{dy}{dx}=\dfrac{e^{ax}.\sec x.\log x}{\sqrt{1-2x}}\left\{a+\tan x+\dfrac{1}{x\log x}+\dfrac{1}{1-2x}\right\}$**.**

**Q91. If** $y=\sin x.\sin 2x.\sin 3x.\sin 4x$**, find** $\dfrac{dy}{dx}$**.**

**(ANS:** $\dfrac{dy}{dx}=\sin x.\sin 2x.\sin 3x.\sin 4x\left[\cot x+2\cot 2x+3\cot 3x+4\cot 4x\right]$ **)**

**Q92. If** $(\sin x)^y = x + y$**, prove that** $\frac{dy}{dx} = \frac{1 - (x + y)\, y \cot x}{(x + y)\log \sin x - 1}$**.**

**Q93. If** $xy \log(x + y) = 1$**, prove that** $\frac{dy}{dx} = -\frac{y(x^2 y + x + y)}{x(xy^2 + x + y)}$**.**

**Q94. Find the derivative of the function** $f(x)$ **given by** $f(x) = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)$ **and hence find** $f^{/}(1)$**.**
**(ANS:** $1 + 2x + 3x^2 + \ldots + 15x^{14}$**,** $f^{/}(1) = 120$**)**

**Q94*: If** $y = \log\frac{x^2 + x + 1}{x^2 - x + 1} + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{1 - x^2}\right)$**, find** $\frac{dy}{dx}$

**(ANS:** $\frac{4}{x^4 + x^2 + 1}$**)**

**DIFFERENTIATION OF INFINITE SERIES**

**Q95. If** $y = x^{x^{x^{x^{\cdots\infty}}}}$ **, find** $\frac{dy}{dx}$**. (ANS:** $\frac{dy}{dx} = \frac{y^2}{x(1 - y\log x)}$**)**

**Q96. If** $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots .to\ \infty}}}$ **, prove that** $\frac{dy}{dx} = \frac{\cos x}{2y - 1}$**.**

**Q97. If** $y = a^{x^{a^{x^{\cdots\infty}}}}$ **, prove that** $\frac{dy}{dx} = \frac{y^2 \log y}{x(1 - y\log x.\log y)}$**.**

**Q98. If** $y = e^{x + e^{x + e^{x + \ldots\ldots to\infty}}}$ **, show that** $\frac{dy}{dx} = \frac{y}{1 - y}$**.**

**Q99. If** $y = \left(\sqrt{x}\right)^{\left(\sqrt{x}\right)^{\left(\sqrt{x}\right)^{\cdots\infty}}}$ **, show that** $\frac{dy}{dx} = \frac{y^2}{x(2 - y\log x)}$**.**

**Q100. If** $y = x + \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x + \ldots.}}}$ **, prove that** $\frac{dy}{dx} = \frac{y}{2y - x}$**.**

**Q101. If** $y = \cfrac{\sin x}{1 + \cfrac{\cos x}{1 + \cfrac{\sin x}{1 + \cfrac{\cos x}{1 + \ldots\infty}}}}$ **, prove that** $\frac{dy}{dx} = \frac{(1 + y)\cos x + y\sin x}{1 + 2y + \cos x - \sin x}$**.**

**Q102. If** $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\infty}}}$ **, prove that** $\frac{dy}{dx} = \frac{1}{2y - 1}$**.**

**Q103. If** $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots\infty}}}$ **, prove that** $(2y - 1)\frac{dy}{dx} = \frac{1}{x}$**.**

**Q104. If** $y = (\sin x)^{(\sin x)^{(\sin x)^{\cdots\infty}}}$ **, prove that** $\frac{dy}{dx} = \frac{y^2 \cot x}{(1 - y\log\sin x)}$**.**

**Q105. If** $y = e^{x^{e^x}} + x^{e^{e^x}} + e^{x^{x^e}}$ **, prove that (ANS:**
$\frac{dy}{dx} = e^{x^{e^x}}.x^{e^x}\left\{\frac{e}{x} + e^x.\log x\right\} + x^{e^{e^x}}.e^{e^x}\left\{\frac{1}{x} + e^x.\log x\right\} + e^{x^{x^e}}.x^{x^e}.x^{e-1}\{1 + e\log x\}$ **)**

**Q106. Prove that the derivative of an even function is an odd function and that of an odd function is an even function.**

**Q107. If** $y = f\left(\frac{2x-1}{x^2+1}\right)$ **and** $f'(x) = \sin x^2$**, find** $\frac{dy}{dx}$**.**

**DIFFERENTIATION OF PARAMETRIC FUNCTIONS**

**Q108. In the following cases, find** $\frac{dy}{dx}$**.**

**(i)** $x = a\left\{\cos t + \frac{1}{2}\log\tan^2\frac{t}{2}\right\}$ **and** $y = a\sin t$**. (ANS:** $\frac{dy}{dx} = \tan t$**)**

**(ii)** $x = a\sec^3\theta$ **and** $y = a\tan^3\theta$ **, find** $\frac{dy}{dx}$ **at** $\theta = \frac{\pi}{3}$**. (ANS:** $\frac{\sqrt{3}}{2}$**)**

**(iii)** $x = a\cos^3 t$ **and** $y = a\sin^3 t$ **(ANS:** $\frac{dy}{dx} = -\tan t$**)**

**(iv)** $x = ae^{\theta}(\sin\theta - \cos\theta)$**,** $y = ae^{\theta}(\sin\theta + \cos\theta)$ **(ANS:** $\cot\theta$ **)**

**(v)** $x = \frac{e^t + e^{-t}}{2}$ **and** $y = \frac{e^t - e^{-t}}{2}$ **(ANS:** $\frac{x}{y}$**)**

**(vi)** $x = \frac{3at}{1+t^2}$ **and** $y = \frac{3at^2}{1+t^2}$ **(ANS:** $\frac{2t}{1-t^2}$**)**

**(vii)** $x = e^{\theta}\left(\theta + \frac{1}{\theta}\right)$ **and** $y = e^{-\theta}\left(\theta - \frac{1}{\theta}\right)$ **(ANS:** $e^{-2\theta}\frac{(\theta^2 - \theta^3 + \theta + 1)}{(\theta^3 + \theta^2 + \theta - 1)}$**)**

**(viii)** $x = \frac{2t}{1+t^2}$ **and** $y = \frac{1-t^2}{1+t^2}$ **(ANS:** $-\frac{x}{y}$**)**

**(ix)** $x = \frac{1-t^2}{1+t^2}$ **and** $y = \frac{2t}{1+t^2}$ **(ANS:** $\frac{t^2-1}{2t}$**)**

**(x) If** $x = 2\cos\theta - \cos 2\theta$ **and** $y = 2\sin\theta - \sin 2\theta$ **, prove that** $\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$

**(xi) If** $x = e^{\cos 2t}$ **and** $y = e^{\sin 2t}$**, prove that** $\frac{dy}{dx} = -\frac{y\log x}{x\log y}$**.**

**(xii) If** $x = a\left(t + \frac{1}{t}\right)$ **and** $y = a\left(t - \frac{1}{t}\right)$**, prove that** $\frac{dy}{dx} = \frac{x}{y}$**.**

## DIFFERENTIATION OF A FUNCTION WITH RESPECT TO ANOTHER FUNCTION

**Q109. Differentiate the following functions w.r.t the given functions:**

**(i)** $\log\sin x$ **w.r.t.** $\sqrt{\cos x}$ **(ANS:** $-2\sqrt{\cos x}.\cot x.\operatorname{cosec} x$**)**

**(ii)** $\tan\left(\frac{1+2x}{1-2x}\right)$ **w.r.t.** $\sqrt{1+4x^2}$ **(ANS:** $\frac{1}{2x\sqrt{1+4x^2}}$**)**

**(iii)** $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ **w.r.t.** $\tan^{-1}x$**,** $x \neq 0$ **(ANS:** $1/2$**)**

**(iv)** $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ **w.r.t.** $\tan^{-1}x$**,** $-1 < x < 1$ **(ANS: 2)**

**(v)** $x^x$ **w.r.t.** $x\log x$ **(ANS:** $x^x$**)**

**(vi)** $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}$ **w.r.t.** $\cos^{-1}x^2$ **(ANS:** $-\frac{1}{2}$**)**

**(vii)** $x^{\sin^{-1}x}$ **w.r.t.** $\sin^{-1}x$ **(ANS:** $x^{\sin^{-1}x}\left\{\log x+\frac{\sqrt{1-x^2}}{x}\sin^{-1}x\right\}$**)**

**(viii)** $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ **w.r.t.** $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ **(ANS: 1)**

**(ix)** $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ **w.r.t.** $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ **(ANS:** $\frac{3}{2}$**)**

**(x)** $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ **w.r.t.** $\sec^{-1}x$ **(ANS:** $\frac{-x\sqrt{x^2-1}}{2}$**)**

**(xi)** $\sin^{-1}\left(2ax\sqrt{1-a^2x^2}\right)$ **w.r.t.** $\sqrt{1-a^2x^2}$**,** $-\frac{1}{\sqrt{2}} ax < \frac{1}{\sqrt{2}}$**. (ANS:** $-\frac{2}{ax}$**)**

**(xii)** $\tan^{-1}\left(\frac{1-x}{1+x}\right)$ **w.r.t.** $\sqrt{1-x^2}$**,** $-1 < x < 1$ **(ANS:** $\frac{\sqrt{1-x^2}}{x\left(1+x^2\right)}$**)**

**(xiii)** $\left(\cos x\right)^{\sin x}$ **w.r.t.** $\left(\sin x\right)^{\cos x}$ **(ANS:** $\frac{\left(\cos x\right)^{\sin x}\left\{\cos x.\log\cos x-\sin x\tan x\right\}}{\left(\sin x\right)^{\cos x}\left\{-\sin x\log\sin x+\cos x.\cot x\right\}}$**)**

## HIGHER ORDER DERIVATIVES

**Q110.** ***If*** $y=\sin^{-1}x$***, show that*** $\frac{d^2y}{dx^2}=\frac{x}{\left(1-x^2\right)^{3/2}}$**.**

**Q111.** ***If*** $y=A\cos nx+B\sin nx$ ***, show that*** $\frac{d^2y}{dx^2}+n^2y=0$

**Q112.** ***If*** $y=A\cos\left(\log x\right)+B\sin\left(\log x\right)$***, prove that*** $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$**.**

**Q113.** ***If*** $y = \tan x + \sec x$***, prove that*** $\frac{d^2y}{dx^2} = \frac{\cos x}{(1-\sin x)^2}$

**Q114.** ***If*** $y = \tan x$***, prove that*** $y_2 = 2yy_1$

**Q115.** ***If*** $y = x\log\left(\frac{x}{a+bx}\right)$***, prove that*** $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$**.**

**Q116.** ***If*** $y = x^x$***, find*** $\frac{d^2y}{dx^2}$ ***(ANS:*** $\frac{d^2y}{dx^2} = x^x\left\{(1+\log x)^2 + \frac{1}{x}\right\}$***)***

**Q117.** ***If*** $y = \log\left\{x + \sqrt{x^2+a^2}\right\}$***, prove that*** $(x^2+a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$**.**

**Q118.** ***If*** $e^y(x+1) = 1$***, show that*** $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$**.**

**Q119.** ***If*** $y = \left\{x + \sqrt{x^2+1}\right\}^m$***, show that*** $(x^2+1)y_2 + xy_1 - m^2y = 0$**.**

**Q120.** ***If*** $y = x^x$***, prove that*** $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$**.**

**Q121.** ***If*** $x = a\cos\theta + b\sin\theta$ ***and*** $y = a\sin\theta - b\cos\theta$ ***, prove that***
$y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$**.**

**Q122.** ***Find*** $\frac{d^2y}{dx^2}$***, if*** $x = at^2$***,*** $y = 2at$**.** ***(ANS:*** $\frac{d^2y}{dx^2} = -\frac{1}{2at^3}$***)***

**Q123.** ***If*** $x = a\cos^3\theta$ ***,*** $y = a\sin^3\theta$ ***, find*** $\frac{d^2y}{dx^2}$***.(ANS:*** $\frac{d^2y}{dx^2} = \frac{1}{3a}\sec^4\theta\,\mathrm{cosec}\theta$ ***)***

**Q124.** ***If*** $x = \tan\left(\frac{1}{a}\log y\right)$***, show that*** $(1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$**.**

## ROLLE'S AND MEAN VALUE THEOREM

**Q125. Discuss the applicability of Rolle's theorem for the following on the indicated intervals:**

**(i)** $f(x) = |x|$ **on** $[-1,1]$ **(ii)** $f(x) = 3 + (x-2)^{2/3}$ **on** $[1,3]$

**(iii)** $f(x) = \tan x$ **on** $[0,\pi]$**. (iv)** $f(x) = \begin{cases} x^2+1, & \text{when } 0 \le x \le 1 \\ 3-x, & \text{when } 1 < x \le 2 \end{cases}$

**(v)** $f(x) = [x]$ **on** $[-1,1]$**, where** $[x]$ **denotes the greatest integer not exceeding 'x'.**

**(vi)** $f(x) = x^{2/3}$ **on** $[-1,1]$**. (vii)** $f(x) = 2x^2 - 5x + 3$ **on** $[1,3]$

**Q126. Verify the Rolle's theorem for the following functions on the given intervals;**

**(i)** $f(x) = x(x-3)^2, 0 \le x \le 3$ **(ANS:** $c = 1$**)**

**(ii)** $f(x) = x^3 - 6x^2 + 11x - 6$ **on the interval** $[1,3]$ **.**

**(iii)** $f(x) = (x-a)^m (x-b)^n$ **on the interval** $[a,b]$ **, where *m, n* are positive integers. (ANS:** $c = \dfrac{mb+na}{m+n}$**)**

**(iv)** $f(x) = \sqrt{4-x^2}$ **on** $[-2,2]$ **(ANS:** $c = 0$**)**

**(v)** $f(x) = \sin^2 x$ **on** $[0,\pi]$ **(ANS:** $c = \pi/2$**)**

**(vi)** $f(x) = \sin x + \cos x - 1$ **on** $[0,\pi/2]$ **(ANS:** $c = \pi/4$**)**

**(vii)** $f(x) = \sin x - \sin 2x$ **on** $[0,\pi]$ **(ANS:** $c = \cos^{-1}\left(\dfrac{1 \pm \sqrt{33}}{8}\right)$**)**

**(viii)** $f(x) = x(x+3)e^{-x/2}$ **on** $[-3,0]$ **(ANS:** $c = -2$**)**

**(ix)** $f(x) = e^x(\sin x - \cos x)$ **on** $[\pi/4, 5\pi/4]$**. (ANS:** $c = \pi$ **)**

**Q127. It is given that for the function** $f(x) = x^3 - 6x^2 + ax + b$ **on** $[1,3]$**, Rolle's Theorem holds with** $c = 2 + \dfrac{1}{\sqrt{3}}$ **.Find the values of a and b, if** $f(1) = f(3) = 0$**. (ANS:** $a = 11$ and $b = -6$**)**

**Q128. It is given that for the function** $f(x) = x^3 + bx^2 + ax$ **on** $[1,3]$**, Rolle's Theorem holds with** $c = 2 + \dfrac{1}{\sqrt{3}}$ **.Find the values of a and b, if** $f(1) = f(3) = 0$**. (ANS:** $a = 11$ and $b = -6$**)**

**Q129. Verify Lagrange's Mean Value theorem for the following functions on the indicated intervals. Also, find a point c in the indicated interval:**

**(i)** $f(x) = x(x-1)(x-2)$ **on** $[0,1/2]$ **(ANS:** $c = 1 - \dfrac{\sqrt{21}}{6}$**)**

**(ii)** $f(x) = x - 2\sin x$ **on** $[-\pi,\pi]$ **(ANS:** $c = \pm(\pi/2)$**)**

**(iii)** $f(x) = 2\sin x + \sin 2x$ **on** $[0,\pi]$ **(ANS:** $c = \pi/3$**)**

**(iv)** $f(x) = \log_e x$ **on** $[1,2]$ **(ANS:** $c = \log_2 e$**)**

## ASSIGNMENT INTEGRATION:

**Evaluate the following integrals**

**1.** $\int \sqrt{1+\sin 2x}\,dx$ **(ANS:** $-\cos x+\sin x+c$ **)**

**2.** $\int \sqrt{1-\sin 2x}\,dx$ **(ANS:** $-\cos x-\sin x+c$ **)**

**3.** $\int \tan^{-1}(\sec x+\tan x)dx,\ -\pi/2 < x < \pi/2$ **(ANS:** $\frac{\pi}{4}x+\frac{x^2}{4}+c$ **)**

**4.** $\int \frac{\left(a^x+b^x\right)^2}{a^x b^x}dx$ **(ANS:** $\frac{\left(a/b\right)^x}{\log_e\left(a/b\right)}+\frac{\left(b/a\right)^x}{\log_e\left(b/a\right)}+2x+c$ **)**

**5.** $\int \frac{e^{5\log_e x}-e^{4\log_e x}}{e^{3\log_e x}-e^{2\log_e x}}dx$ **(ANS:** $\frac{x^3}{3}+c$ **)**

**6.** $\int \frac{\cos x-\cos 2x}{1-\cos x}dx$ **(ANS:** $2\sin x+x+c$ **)**

**7.** $\int \frac{\sin^6 x+\cos^6 x}{\sin^2 x\cos^2 x}dx$ **(ANS:** $\tan x-\cot x-3x+c$ **)**

**8.** $\int \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}dx,\ -\pi/2 < x < \pi/2$ **(ANS:** $\frac{\pi}{4}x-\frac{x^2}{2}+c$ **)**

**9.** $\int \frac{x^2}{(a+bx)^2}dx$ **(ANS:** $\frac{1}{b^3}\left\{bx-2a\log|bx+a|-\frac{a^2}{(a+bx)}\right\}+c$ **)**

**10.** $\int \sqrt{1\mp\sin x}\,dx$ **(ANS:** $2\left(\sin\frac{x}{2}\pm\cos\frac{x}{2}\right)+c$ **)**

**11.** $\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx$ **(ANS:** $-\frac{1}{2}\sin 2x+c$ **)**

**12.** $\int e^{3\log x}\left(x^4+1\right)^{-1}dx$ **(ANS:** $\frac{1}{4}\log\left(x^4+1\right)+c$ **)**

**13.** $\int \frac{1}{\sin(x-a)\cos(x-b)}dx$ **(ANS:** $\frac{1}{\cos(a-b)}\log_e\left|\frac{\sin(x-a)}{\cos(x-b)}\right|+c$ **)**

**14.** $\int \tan x\tan 2x\tan 3x\,dx$ **(ANS:** $-\frac{1}{3}\log|\cos 3x|+\frac{1}{2}\log|\cos 2x|+\log|\cos x|+c$ **)**

**15.** $\int \frac{\tan x}{a+b\tan^2 x}dx$ **(ANS:** $\frac{1}{\left(a^2-b^2\right)}\log\left|a^2\sin^2 x+b^2\cos^2 x\right|+c$ **)**

**16.** $\int \frac{\sin(x+a)}{\sin(x+b)}dx$ **(ANS:** $(x+b)\cos(a-b)+\sin(a-b)\log|\sin(x+b)|+c$ **)**

**17.** $\int \left[1+2\tan x(\tan x+\sec x)\right]^{1/2}dx$ **(ANS:** $\log|\sec x+\tan x|+\log|\sec x|+c$ **)**

**18.** $\int \frac{\sec x\,\mathrm{cosec}\,x}{\log(\tan x)}dx$ **(ANS:** $\log(\log\tan x)+C$ **)**

**19.** $\int \frac{\sec x}{\log(\sec x+\tan x)}dx$ **(ANS:** $\log\left|\log(\sec x+\tan x)\right|+C$**)**

**20.** $\int \frac{e^{x-1}+x^{e-1}}{e^{x}+x^{e}}dx$ **(ANS:** $\frac{1}{e}\log\left|e^{x}+x^{e}\right|+C$**)**

**21.** $\int \frac{\sin 2x}{\sin\left(x-\frac{\pi}{6}\right).\sin\left(x+\frac{\pi}{6}\right)}dx$ **(ANS:** $\log\left|\sin\left(x-\frac{\pi}{6}\right)\sin\left(x+\frac{\pi}{6}\right)\right|+C$**)**

**22.** $\int 2^{2^{2x}}2^{2^{x}}2^{x}dx$ **(ANS:** $\frac{1}{(\log 2)^{3}}2^{2^{2^{x}}}+C$**)**

**23.** $\int \sec x\log(\sec x+\tan x)dx$ **(ANS:** $\frac{1}{2}\{\log|\sec x+\tan x|\}^{2}+C$**)**

**24.** $\int \cos^{3}x\, e^{\log\sin x}dx$ **(ANS:** $-\frac{\cos^{4}x}{4}+C$**)**

**25.** $\int 5^{x+\tan^{-1}x}.\left(\frac{x^{2}+2}{x^{2}+1}\right)dx$ **(ANS:** $\frac{1}{\log 5}\left(5^{x+\tan^{-1}x}\right)+C$**)**

**26.** $\int \frac{x}{\sqrt{x^{2}+a^{2}}+\sqrt{x^{2}-a^{2}}}dx$ **(ANS:** $\frac{1}{6a^{2}}\left\{\left(x^{2}+a^{2}\right)^{3/2}-\left(x^{2}-a^{2}\right)^{3/2}\right\}+C$**)**

**27.** $\int \frac{e^{x}}{e^{2x}+6e^{x}+5}dx$ **(ANS:** $\frac{1}{4}\log\left|\frac{e^{x}+1}{e^{x}+5}\right|+C$**)**

**28.** $\int \frac{2x^{3}}{4+x^{8}}dx$ **(ANS:** $\frac{1}{4}\tan^{-1}\left(\frac{x^{4}}{2}\right)+C$**)**

**29.** $\int \frac{1}{x\left(x^{6}+1\right)}dx$ **(ANS:** $\frac{1}{6}\log\left|\frac{x^{6}}{x^{6}+1}\right|+C$**)**

**30.** $\int \frac{x}{3x^{4}-18x^{2}+11}dx$ **(ANS:** $\frac{\sqrt{3}}{48}\log\left|\frac{x^{2}-3-\frac{4}{\sqrt{3}}}{x^{2}-3+\frac{4}{\sqrt{3}}}\right|+C$**)**

**31.** $\int \frac{a^{x}}{\sqrt{1-a^{2x}}}dx$ **(ANS:** $\frac{1}{\log a}\sin^{-1}\left(a^{x}\right)+C$**)**

**32.** $\int \sqrt{\frac{x}{a^{3}-x^{3}}}dx$ **(ANS:** $\frac{2}{3}\sin^{-1}\left(\frac{x^{3/2}}{a^{3/2}}\right)+C$**)**

**33.** $\int \sqrt{\sec x-1}\,dx$ **(ANS:** $-\log\left|\left(\cos x+\frac{1}{2}\right)+\sqrt{\cos^{2}x+\cos x}\right|+C$**)**

**34.** $\int \sqrt{\cos ecx-1}\,dx$ **(ANS:** $\log\left|\left(\sin x+\frac{1}{2}\right)+\sqrt{\sin^{2}x+\sin x}\right|+C$**)**

**35.** $\int \frac{1}{x^{2/3}\sqrt{x^{2/3}-4}}dx$ **(ANS:** $3\log\left|x^{1/3}+\sqrt{x^{2/3}-4}\right|+C$ **)**

**36.** $\int \frac{ax^3+bx}{x^4+c^2}dx$ **(ANS:** $\frac{a}{4}\log\left|x^4+c^2\right|+\frac{b}{2c}\tan^{-1}\left(\frac{x^2}{c}\right)+C$ **)**

**37.** $\int \sqrt{\frac{1+x}{x}}dx$ **(ANS:** $\sqrt{x^2+x}+\frac{1}{2}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x}\right|+C$ **)**

**38.** $\int \sqrt{\frac{a-x}{a+x}}dx$ **(ANS:** $a\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{a^2-x^2}+C$ **)**

**39.** $\int x\sqrt{\frac{a^2-x^2}{a^2+x^2}}dx$ **(ANS:** $\frac{1}{2}a^2\sin^{-1}\left(\frac{x^2}{a^2}\right)+\frac{1}{2}\sqrt{a^4-x^4}+C$ **)**

**40.** $\int \sqrt{\frac{1-x}{1+x}}dx$ **(ANS:** $\sin^{-1}x+\sqrt{1-x^2}+C$ **)**

**41.** $\int \frac{\sin x}{\sin 3x}dx$ **(ANS:** $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}\right|+C$ **)**

**42.** $\int \frac{1}{3+\sin 2x}dx$ **(ANS:** $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{3\tan x+1}{2\sqrt{2}}\right)+C$ **)**

**43.** $\int \frac{1}{2-3\cos 2x}dx$ **(ANS:** $\frac{1}{2\sqrt{5}}\log\left|\frac{\sqrt{5}\tan x-1}{\sqrt{5}\tan x+1}\right|+C$ **)**

**44.** $\int \frac{\cos x}{\cos 3x}dx$ **(ANS:** $\frac{1}{2\sqrt{3}}\log\left|\frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x}\right|+C$ **)**

**45.** $\int \frac{1}{(\sin x-2\cos x)(2\sin x+\cos x)}dx$ **(ANS:** $\frac{1}{5}\log\left|\frac{\tan x-2}{2\tan x+1}\right|+C$ **)**

**46.** $\int \frac{\sin 2x}{\sin^4 x+\cos^4 x}dx$ **(ANS:** $\tan^{-1}\left(\tan^2 x\right)+C$ **)**

**47.** $\int \frac{1}{\sin^2 x+\sin 2x}dx$ **(ANS:** $\frac{1}{2}\log\left|\frac{\tan x}{\tan x+2}\right|+C$ **)**

**48.** $\int \frac{1+\sin x}{\sin x(1+\cos x)}dx$ **(ANS:** $\frac{1}{2}\left\{\log\left|\tan x/2\right|+\frac{\tan^2 x/2}{2}+2\tan x/2\right\}+C$ **)**

**49.** $\int \frac{3\cos x+2}{\sin x+2\cos x+3}dx$ **(ANS:** $\frac{6x}{5}+\frac{3}{5}\log\left|\sin x+2\cos x+3\right|-\frac{8}{5}\tan^{-1}\left(\frac{\tan\frac{x}{2}+1}{2}\right)+C$ **)**

**50.** $\int \frac{8\cot x+1}{3\cot x+2}dx$ **(ANS:** $2x+\log\left|2\sin x+3\cos x\right|+C$ **)**

**51.** $\int \frac{1}{p+q\tan x}dx$ **(ANS:** $\frac{p}{p^2+q^2}x+\frac{q}{p^2+q^2}\log\left|p\cos x+q\sin x\right|+C$ **)**

**52.** $\int \frac{x - \sin x}{1 - \cos x} dx$ **(ANS:** $-x\cot\frac{x}{2} + C$**)**

**53.** $\int x^3 e^x dx$ **(ANS:** $\left(x^3 - 3x^2 + 6x - 6\right)e^x + C$**)**

**54.** $\int x\sin x\cos x dx$ **(ANS:** $-\frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x + C$**)**

**55.** $\int \left(\log x\right)^2 .x dx$ **(ANS:** $\frac{x^2}{2}\left[\left(\log x\right)^2 - \log x + \frac{1}{2}\right] + C$**)**

**56.** $\int \frac{\log\left(x+2\right)}{\left(x+2\right)^2} dx$ **(ANS:** $-\frac{1}{x+2} - \frac{\log\left(x+2\right)}{\left(x+2\right)} + C$**)**

**57.** $\int x\left(\frac{\sec 2x - 1}{\sec 2x + 1}\right) dx$ **(ANS:** $x\tan x - \log\sec x - \frac{x}{2} + C$**)**

**58.** $\int \sin^{-1}\left(3x - 4x^3\right) dx$ **(ANS:** $3x\sin^{-1} x + 3\sqrt{1 - x^2} + C$**)**

**59.** $\int \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) dx$ **(ANS:** $3x\tan^{-1} x - \frac{3}{2}\log\left|x^2 + 1\right| + C$**)**

**60.** $\int \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right) dx$ **(ANS:** $2x\tan^{-1} x - \log\left|1 + x^2\right| + C$**)**

**61.** $\int \tan^{-1}\sqrt{\frac{1 - x}{1 + x}} dx$ **(ANS:** $\frac{1}{2}x\left(\cos^{-1} x\right) - \frac{1}{2}\sqrt{1 - x^2} + C$**)**

**62.** $\int \sin^{-1}\sqrt{\frac{x}{a + x}} dx$ **(ANS:** $x\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{ax} + a\tan^{-1}\sqrt{\frac{x}{a}} + C$**)**

**63.** $\int \frac{\left(x\tan^{-1} x\right)}{\left(1 + x^2\right)^{3/2}} dx$ **(ANS:** $-\frac{\tan^{-1} x}{\sqrt{1 + x^2}} + \frac{x}{\sqrt{1 + x^2}} + C$**)**

**64.** $\int \frac{x^3\sin^{-1} x^2}{\sqrt{1 - x^4}} dx$ **(ANS:** $\frac{1}{2}\left[x^2 - \sqrt{1 - x^4}\sin^{-1} x^2\right] + C$**)**

**65.** $\int \tan^{-1}\left(\sqrt{x}\right) dx$ **(ANS:** $\left(x + 1\right)\tan^{-1}\sqrt{x} - \sqrt{x} + C$**)**

**66.** $\int \sec^{-1}\sqrt{x} dx$ **(ANS:** $x\sec^{-1}\sqrt{x} - \sqrt{x - 1} + C$**)**

**67.** $\int e^x\left(\frac{2 + \sin 2x}{1 + \cos 2x}\right) dx$ **(ANS:** $e^x\tan x + C$**)**

**68.** $\int e^x\left(\frac{1 - \sin x}{1 - \cos x}\right) dx$ **(ANS:** $-e^x\cot\frac{x}{2} + C$**)**

**69.** $\int \left\{\sin\left(\log x\right) + \cos\left(\log x\right)\right\} dx$ **(ANS:** $x\sin\left(\log x\right) + C$**)**

**70.** $\int \frac{\log x}{\left(1 + \log x\right)^2} dx$ **(ANS:** $\frac{x}{\left(\log x + 1\right)} + C$**)**

**71.** $\int e^{2x}\left(\frac{1 + \sin x\cos x}{\cos^2 x}\right) dx$ **(ANS:** $e^x\tan x + C$**)**

**72.** $\int e^x\left(\frac{x-1}{2x^2}\right)dx$ **(** $\frac{e^x}{2x}+C$ **)**

**73.** $\int e^x\frac{x-1}{(x+1)^3}dx$ **(ANS:** $\frac{e^x}{(x+1)^2}+C$ **)**

**74.** $\int e^x\frac{(1-x)^2}{(1+x^2)^2}dx$ **(ANS:** $\frac{e^x}{1+x^2}+C$ **)**

**75.** $\int e^x\left(\frac{\sin 4x-4}{1-\cos 4x}\right)dx$ **(ANS:** $e^x\cot 2x+C$ **)**

**76.** $\int \frac{\sqrt{1-\sin x}}{1+\cos x}e^{-x/2}dx$ **(ANS:** $-e^{-x/2}\sec(x/2)+C$ **)**

**77.** $\int \frac{e^x}{x}\left\{x(\log x)^2+2\log x\right\}dx$ **(ANS:** $e^x(\log x)^2+C$ **)**

**78.** $\int e^x\frac{\sqrt{1-x^2}\sin^{-1}x+1}{\sqrt{1-x^2}}dx$ **(ANS:** $e^x\sin^{-1}x+C$ **)**

**79.** $\int\left(\frac{1}{\log x}-\frac{1}{(\log x)^2}\right)dx$ **(ANS:** $\frac{x}{\log x}+C$ **)**

**80.** $\int\left\{\tan(\log x)+\sec^2(\log x)\right\}dx$ **(ANS:** $x\tan(\log x)+C$ **)**

**81.** $\int \sin(\log x)dx$ **(ANS:** $\frac{x}{2}\left\{\sin(\log x)-\cos(\log x)\right\}+C$ **)**

**82.** $\int e^x\cos^2 x\,\mathrm{dx}$ **(ANS:** $\frac{1}{2}e^x+\frac{e^x}{10}(\cos 2x+2\sin 2x)+C$ **)**

**83.** $\int \cos(\log x)\,dx$ **(ANS:** $\frac{x}{2}(\cos(\log x)+\sin(\log x))+C$ **)**

**84..** $\int \frac{1}{x^3}\sin(\log x)dx$ **(ANS:** $-\frac{1}{5x^2}\left[\cos(\log x)+2\sin(\log x)\right]+C$ **)**

**85.** $\int \frac{2x}{x^3-1}dx$ **(ANS:** $\frac{2}{3}\log|x-1|-\frac{1}{3}\log\left|x^2+x+1\right|+\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+C$ **)**

**86.** $\int \frac{1}{\sin x-\sin 2x}dx$ **(ANS:** $-\frac{1}{2}\log|1-\cos x|-\frac{1}{6}\log|1+\cos x|+\frac{2}{3}\log|1-2\cos x|+C$ **)**

**87.** $\int \frac{1-\cos x}{\cos x(1+\cos x)}dx$ **(ANS:** $\log|\sec x+\tan x|-2\tan x/2+C$ **)**

**88.** $\int \frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)}dx$ **(ANS:** $x+3\log|x-4|-24\log|x-5|+30\log|x-6|+C$ **)**

**89.** $\int \frac{\tan\theta+\tan^3\theta}{1+\tan\theta}d\theta$ **(ANS:** $-\frac{1}{3}\log|1+\tan\theta|+\frac{1}{6}\log\left|\tan^2\theta-\tan\theta+1\right|+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan\theta-1}{\sqrt{3}}\right)+C$ **)**

**90.** $\int \frac{\sin x}{\sin 4x} dx$ **(ANS:** $-\frac{1}{8}\log\left|\frac{1+\sin x}{1-\sin x}\right| + \frac{1}{4\sqrt{2}}\log\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right| + C$ **)**

**91.** $\int \frac{x}{\left(x^2-a^2\right)\left(x^2-b^2\right)} dx$ **(ANS:** $\frac{1}{2\left(a^2-b^2\right)}\log\left|\frac{x^2-a^2}{x^2-b^2}\right| + C$ **)**

**92.** $\int \frac{x^2+x-1}{\left(x+1\right)^2\left(x+2\right)} dx$ **(ANS:** $\frac{1}{x+1} + \log\left|x+2\right| + C$ **)**

**93.** $\int \frac{5x^2+20x+6}{x^3+2x^2+x} dx$ **(ANS:** $6\log\left|x\right| - \log\left|x+1\right| + \frac{9}{\left(x+1\right)} + C$ **)**

**94.** $\int \frac{4x^4+3}{\left(x^2+2\right)\left(x^2+3\right)\left(x^2+4\right)} dx$ **(ANS:** $\frac{19}{2\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{39}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + \frac{67}{4}\tan^{-1}\left(\frac{x}{2}\right) + C$ **)**

**95.** $\int \frac{1}{\left(x-3\right)\sqrt{x+1}} dx$ **(ANS:** $2\left[\sqrt{x} - \tan^{-1}\sqrt{x}\right] + C$ **)**

**96.** $\int \frac{1}{\left(x^2-4\right)\sqrt{x+1}} dx$ **(ANS:** $\frac{1}{4\sqrt{3}}\log\left|\frac{\sqrt{x+1}-\sqrt{3}}{\sqrt{x+1}+\sqrt{3}}\right| - \frac{1}{2}\tan^{-1}\left(\sqrt{x+1}\right) + C$ **)**

**97.** $\int \frac{x+2}{\left(x^2+3x+3\right)\sqrt{x+1}} dx$ **(ANS:** $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3\left(x+1\right)}}\right) + C$ **)**

**98.** $\int \frac{1}{\left(x+1\right)\sqrt{x^2-1}} dx$ **(ANS:** $\sqrt{\frac{x-1}{x+1}} + C$ **)**

**99.** $\int \frac{1}{\left(x-1\right)\sqrt{x^2+4}} dx$ **(ANS:** $-\frac{1}{\sqrt{5}}\log\left|\frac{1}{x-1} + \frac{1}{5} + \sqrt{\frac{x^2+4}{5\left(x-1\right)^2}}\right| + C$ **)**

**100.** $\int \frac{1}{x^2\sqrt{1+x^2}} dx$ **(ANS:** $-\frac{\sqrt{1+x^2}}{x} + C$ **)**

**101.** $\int \frac{\sqrt{1+x^2}}{1-x^2} dx$ **(ANS:** $-\frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{1+x^2}-\sqrt{2}x}{\sqrt{1+x^2}+\sqrt{2}x}\right| - \log\left|x+\sqrt{1+x^2}\right| + C$ **)**

**102.** $\int \frac{1}{x\sqrt{ax-x^2}} dx$ **(ANS:** $\frac{-2}{a}\sqrt{\frac{a-x}{x}} + C$ **)**